Analysis of a Microstrip Crossover Embedded in a Multilayered Anisotropic and Lossy Media

Jesús Martel, Rafael R. Boix and Manuel Horno, Member, IEEE

Abstract—The equivalent circuit and the scattering parameters of the orthogonal microstrip crossover discontinuity are determined by assuming that the conducting strips are embedded in a multilayered substrate which may contain both anisotropic dielectrics and materials with a nonnegligible conductivity. The equivalent circuit of the crossover is obtained in terms of the complex excess charge densities on the strips. These excess charge densities are computed by means of the Galerkin method in the spectral domain. Comparison is carried out with previously existing results for microstrip crossovers on lossless isotropic substrates and original results are presented for crossovers on anisotropic and lossy substrates.

I. INTRODUCTION

In digital integrated circuits which are designed with a multilevel metallization scheme, crossings take place between interconnection conducting strips. The coupling between crossing strips can be significant (specially, in high speed circuits), and it can be the origin of circuit delay, distortion and crosstalk [1]. In microwave integrated circuits, air-bridge and underpass crossings between conducting strips can be found in components such as interdigitated couplers [2], square spiral inductors [3], square spiral transformers [4] and filters [5]. In all these cases, the coupling effects coming from strip crossings are also noticeable since the vertical distance between crossing conductors is usually very small [6].

Many works have been published in the literature in relation to the modelling of the coupling between noncoplanar crossing conductors. Thus, the equivalent lumped circuit for the crossing at an arbitrary angle of two cylindrical wires placed in free space has been obtained in [7]. Papatheodorou et al. have determined the capacitive equivalent circuit of the orthogonal crossing between two nonintersecting conducting strips in a two-dielectric medium [8]. The same authors have also carried out the fullwave analysis of the strip crossing structure in the case in which the conducting strips are placed inside a homogeneous medium [9]. Other configurations of orthogonal strip crossings, involving a multilayered substrate with three different media, have been characterized in [6], [10], [11]. In spite of the abundant bibliography existing on noncoplanar crossing conductors, in all the papers mentioned in this paragraph, the authors have always assumed that the media of the conductors substrate have a vanishing conductivity, thus excluding doped semiconductors from their analyses.

In this paper, the equivalent circuit and the scattering parameters of a microstrip crossover are obtained on the assumption that the conducting strips of the crossover are embedded in a stratified substrate containing an arbitrary number of layers of lossy dielectric materials. Layers of doped semiconductors with a nonnegligible conductivity are allowed in the analysis carried out. This has a practical interest since doped semiconductors appear in integrated circuits as a part of active devices and they can also be incorporated in microwave integrated circuits for the realization of slow-wave planar transmission lines [12]. Materials with dielectric anisotropy are also allowed in the scope of the current paper so that some materials used in the manufacture of microwave integrated circuits, such as sapphire and epsilam-10 [13], can be adequately taken into account in the analysis. Given the fact that the substrate of the microstrip crossover analyzed in this paper consists of lossy materials, both capacitive ([8], [11]) and resistive components must be at least included in the equivalent circuit of the discontinuity.

The determination of the equivalent circuit of the microstrip crossover has been carried out in the current paper by means of the excess charge technique in the spectral domain. This technique has been used by the authors to characterize other microstrip discontinuities in the range of frequency for which the static models of microstrip discontinuities are valid [14]. When applying the excess charge technique to the analysis of the microstrip crossover, lossy materials have been treated as dielectric materials with a complex permittivity. As it happens with the quasi-TEM analysis of planar transmission lines on lossy substrates [15], the excess charge technique, which is basically an electrostatic technique, is expected to be correct for microstrip discontinuities on lossy substrates when certain conditions are fulfilled. These conditions are essentially that the dimensions of the discontinuities must be much smaller than the lines wavelength [9] and the substrates skin depth, and that the conductivities of the substrates must not be too high [16]. It has been checked that the results presented at the end of the paper for microstrip crossovers on lossy substrates correspond to frequency values, conductivities values and dimensions values which are within the limits that have been already proved to be valid for an electrostatic type of analysis by comparing with full wave approaches [9], [16].

II. STATEMENT OF THE PROBLEM AND ANALYSIS

In Fig. 1(a), the cross section of a multilayered medium...
is shown, which acts as a substrate for the two conducting strips of the orthogonal microstrip crossover analyzed in the current paper. Each layer of the substrate is allowed to have nonnegligible conductivity and/or uniaxial dielectric anisotropy, the optical axis being aligned with the y-axis of Fig. 1(a) in the latter case. The physical properties assumed for the multilayered substrate are incorporated in the analysis by assigning frequency-dependent permittivity tensors to the layers, which are given by

$$
\epsilon_i = \epsilon_0 \left( \begin{array}{ccc}
\epsilon_{xx,i} - \frac{\sigma_{xx,i}}{2\pi f \epsilon_0} & 0 & 0 \\
0 & \epsilon_{yy,i} - \frac{\sigma_{yy,i}}{2\pi f \epsilon_0} & 0 \\
0 & 0 & \epsilon_{zz,i} - \frac{\sigma_{zz,i}}{2\pi f \epsilon_0}
\end{array} \right)
$$

where $$\epsilon_i(i = 1, \ldots, N_3)$$ is the actual permittivity tensor of the i-th layer, $$\sigma_i(i = 1, \ldots, N_3)$$ is the conductivity of the i-th layer, $$\hat{f}$$ is the 3 x 3 unit matrix and $$f$$ is the frequency. The conducting strips of the orthogonal microstrip crossover are placed at the interfaces $$y = H_N$$ and $$y = H_N$$ of the multilayered substrate (see Fig. 1(a)), their position with respect to the coordinate axes being indicated in Fig. 1(b). These conducting strips are assumed to be lossless, infinitely thin and infinitely long.

In Fig. 1(c), the low-frequency equivalent circuit of the microstrip crossover appearing in Figs. 1(a) and 1(b) is depicted. The reference planes $$T_1$$ and $$T_2$$ are taken to be coincident with the axial planes of the conducting strips ($$x = 0$$ and $$z = 0$$ in Fig. 1(b). The quantities $$Z_1$$ and $$Z_2$$ are the impedances of the microstrip lines crossing each other, which are complex when the conductivities of the layers in Fig. 1(a) are not negligible [16]. The capacitors of the equivalent circuit shown in Fig. 1(c) account for the distortion experienced in the vicinity of the crossover by the electric field existing far from the crossover, around the microstrip lines. The resistors of the equivalent circuit account for the distortion experienced by the conduction currents existing far from the crossover inside the layers with nonvanishing conductivity. The scattering parameters of the four-port network of Fig. 1(c) at the reference planes $$T_1$$ and $$T_2$$ can be obtained in terms of the impedances $$Z_1$$ and $$Z_2$$, the capacitances $$C_{p1}$$, $$C_{p2}$$ and $$C_s$$, and the conductances $$G_{p1}$$, $$G_{p2}$$ and $$G_s$$. In the calculation of the capacitances and conductances of the lumped circuit elements shown in Fig. 1(c), the excess-charge technique in the spectral domain has been used. The steps followed in the application of the technique are similar to those exposed in [14], the main difference being that in this paper some physical quantities have turned out to be complex and frequency-dependent owing to the fact that the permittivity tensors of the layers of the multilayered substrate (Fig. 1(a)) have also been taken to be complex and frequency-dependent (see (1)).

Let $$V_1$$ ($$V_2$$) be the electrostatic excitation potential of the conducting strip of width $$w_1$$ ($$w_2$$) shown in Fig. 1(a) and (b), and let $$\sigma^1(x,z)$$ ($$\sigma^2(x,z)$$) be the surface charge density on the same strips. The functions $$\sigma^1(x,z)$$ and $$\sigma^2(x,z)$$ can be split into two terms as follows:

$$
\sigma^1(x,z) = \sigma_{\infty}^1(z) + \sigma_{n=1}^1(x,z)\{|x| < \infty; |z| \leq w_1/2\} \tag{2}
$$

$$
\sigma^2(x,z) = \sigma_{\infty}^2(x) + \sigma_{n=1}^2(x,z)\{|x| \leq w_2/2; |z| < \infty\} \tag{3}
$$

where $$\sigma_{\infty}^1(z)$$ ($$\sigma_{\infty}^2(x)$$) stands for the surface charge density that would appear on the infinitely long strip of width $$w_1$$ ($$w_2$$) if the strip of width $$w_1$$ ($$w_2$$) were taken out of the
multilayered substrate, and \( \sigma_{ex}(x, z) \) stands for the excess charge density that exists on the strip of width \( w_1 \) (\( w_2 \)) in the neighborhood of the crossover. Assuming that \( \sigma_{ex}^{1}(z) \) and \( \sigma_{ex}^{2}(z) \) are known functions that have been obtained by solving the related two-dimensional problems [15], it can be shown [8] that the functions \( \sigma_{ex}^{1}(x, z) \) and \( \sigma_{ex}^{2}(x, z) \) are the solution of the following set of integral equations:

\[
V_1 = \int_{-w_1/2}^{w_1/2} \int_{-\infty}^{\infty} G(x-x', y = H_{Ni}, y' = H_{Ni}, z = z') \times \left[ \sigma_{ex}^{1}(x', z') + \sigma_{ex}^{2}(x', z') \right] \, dx' \, dz',
\]

\[
+ \int_{-\infty}^{+\infty} \int_{-w_2/2}^{+w_2/2} G(x - x', y = H_{Ni}, y' = H_{Ni}, z = z') \times \left[ \sigma_{ex}^{1}(x', z') + \sigma_{ex}^{2}(x', z') \right] \, dx' \, dz',
\]

\[
|z| < \infty; \ |x| \leq w_1/2
\]

(4)

\[
V_2 = \int_{-w_1/2}^{w_1/2} \int_{-\infty}^{\infty} G(x-x', y = H_{Ni}, y' = H_{Ni}, z = z') \times \left[ \sigma_{ex}^{1}(x', z') + \sigma_{ex}^{2}(x', z') \right] \, dx' \, dz',
\]

\[
+ \int_{-\infty}^{+\infty} \int_{-w_2/2}^{+w_2/2} G(x - x', y = H_{Ni}, y' = H_{Ni}, z = z') \times \left[ \sigma_{ex}^{1}(x', z') + \sigma_{ex}^{2}(x', z') \right] \, dx' \, dz',
\]

\[
|z| \leq w_2/2; \ |x| < \infty
\]

(5)

where \( G(x-x', y, y', z-z') \) stands for the three-dimensional complex electrostatic Green’s function of the multilayered lossy substrate shown in Fig. 1(a). It can be proved that the capacitances and conductances of the lumped circuit elements shown in Fig. 1(c) can be easily obtained in terms of the complex functions \( \sigma_{ex}^{i}(x, z) \) (\( i = 1, 2 \)) as follows:

\[
C_p1 = \frac{G_{p1}}{2\pi f} = \frac{1}{V_1} \left[ \int_{-w_1/2}^{w_1/2} \int_{-\infty}^{\infty} \sigma_{ex}^{1}(x, z) \, dx dz \right] \bigg|_{V_1=V_2}
\]

(6)

\[
C_p2 = \frac{G_{p2}}{2\pi f} = \frac{1}{V_2} \left[ \int_{-\infty}^{+\infty} \int_{-w_2/2}^{+w_2/2} \sigma_{ex}^{1}(x, z) \, dx dz \right] \bigg|_{V_1=V_2}
\]

(7)

\[
C_s = \frac{G_s}{2\pi f} = \frac{1}{2} \left[ \frac{1}{V_1} \left[ \int_{-w_1/2}^{w_1/2} \int_{-\infty}^{\infty} \sigma_{ex}^{1}(x, z) \, dx dz \right] \bigg|_{V_1=-V_2} \right.
\]

\[
- \frac{1}{V_1} \left[ \int_{-\infty}^{+\infty} \int_{-w_2/2}^{+w_2/2} \sigma_{ex}^{1}(x, z) \, dx dz \right] \bigg|_{V_1=-V_2}
\]

\[
- \left. \frac{1}{V_2} \left[ \int_{-w_1/2}^{w_1/2} \int_{-\infty}^{\infty} \sigma_{ex}^{2}(x, z) \, dx dz \right] \bigg|_{V_1=-V_2} \right.
\]

\[
- \frac{1}{V_2} \left[ \int_{-\infty}^{+\infty} \int_{-w_2/2}^{+w_2/2} \sigma_{ex}^{2}(x, z) \, dx dz \right] \bigg|_{V_1=-V_2}
\]

\[
= \frac{G_{p1} + G_{p2}}{2\pi f}
\]

(8)

These three expressions indicate that the determination of all the circuit parameters of the crossover discontinuity analyzed in the current paper reduces to calculating the complex excess charge densities on the conducting strips of the discontinuity for two different potential excitations: the quasi-even excitation in which \( V_1 = V_2 \) and the quasi-odd excitation, in which \( V_1 = -V_2 \). Instead of calculating the excess charge densities, in this paper, we have focused on calculating their two-dimensional Fourier-transform in quasi-even and quasi-odd excitations by using a spectral version of the set of integral equations (4) and (5).

Let \( \phi_1(\alpha, y = H_{Ni}, \beta) \) and \( \phi_2(\alpha, y = H_{Ni}, \beta) \) be the two-dimensional Fourier transform of the electrostatic potential at the interfaces \( y = H_{Ni} \) and \( y = H_{Ni} \) of the multilayered substrate shown in Fig. 1(a), let \( G(\alpha, y = H_{Ni}, y' = H_{Ni}, \beta) \) (\( i, j = 1, 2 \)) be the two-dimensional Fourier transform of the functions \( G(x, y = H_{Ni}, y' = H_{Ni}, z) \) (\( i, j = 1, 2 \)) introduced in (4) and (5), and let 2\( \pi \delta(\alpha) \sigma_{ex}^{1}(\beta) \), 2\( \pi \sigma_{ex}^{2}(\alpha) \delta(\beta) \) and \( \sigma_{ex}^{i}(\alpha, \beta) \) (\( i = 1, 2 \)) be respectively the two-dimensional Fourier transform of the functions \( \sigma_{ex}^{1}(z) \), \( \sigma_{ex}^{2}(x) \) and \( \sigma_{ex}^{i}(x, z) \) (\( i = 1, 2 \)), such as they appear in (2) and (3) \( \delta(\cdot) \) stands for the Dirac delta. There are two-algebraic relations among all the spectral functions that have just been defined. These algebraic relations are given by

\[
\phi_1(\alpha, y = H_{Ni}, \beta) = G(\alpha, y = H_{Ni}, y' = H_{Ni}, \beta)
\]

\[
\times \left[ 2\pi \delta(\alpha) \sigma_{ex}^{1}(\beta) + \sigma_{ex}^{1}(\alpha, \beta) \right]
\]

\[
+ G(\alpha, y = H_{Ni}, y' = H_{Ni}, \beta) \times 2\pi \sigma_{ex}^{2}(\alpha) \delta(\beta) + \sigma_{ex}^{2}(\alpha, \beta) \right] \)

(9)

\[
\phi_2(\alpha, y = H_{Ni}, \beta) = G(\alpha, y = H_{Ni}, y' = H_{Ni}, \beta)
\]

\[
\times 2\pi \delta(\alpha) \sigma_{ex}^{1}(\beta) + \sigma_{ex}^{1}(\alpha, \beta)
\]

\[
+ G(\alpha, y = H_{Ni}, y' = H_{Ni}, \beta) \times 2\pi \sigma_{ex}^{2}(\alpha) \delta(\beta) + \sigma_{ex}^{2}(\alpha, \beta) \right] \)

(10)

In the above mathematical relations, it can be seen that algebraic products take the place of the equivalent convolution integrals appearing in (4) and (5).

The functions \( G(\alpha, y = H_{Ni}, y' = H_{Ni}, \beta) \) (\( i, j = 1, 2 \)) shown in (9) and (10) can be calculated for the generic multilayered substrate of Fig. 1(a) by using the recurrent algorithms described in the Appendix. Also, the functions \( \sigma_{ex}^{1}(\beta) \) and \( \sigma_{ex}^{2}(\alpha) \) can be obtained by applying the Galerkin method in the spectral domain to the related two-dimensional problems as explained in [15]. Since adequate methods have been found to calculate \( \sigma_{ex}^{1}(\beta) = G(\alpha, y = H_{Ni}, y' = H_{Ni}, \beta) \) (\( i = 1, 2 \)), \( \sigma_{ex}^{1}(\alpha) \) and \( \sigma_{ex}^{2}(\alpha) \), and in order to determine the two-dimensional Fourier transforms of the excess charge densities, \( \sigma_{ex}^{1}(\alpha, \beta) \) and \( \sigma_{ex}^{2}(\alpha, \beta) \), in this work we have applied the Galerkin method in the spectral domain to the (9) and (10) as described in [14]. After calculating the values of \( \sigma_{ex}^{1}(\alpha, \beta) \) and \( \sigma_{ex}^{2}(\alpha, \beta) \) for quasi-even and quasi-odd excitations of the crossing strips shown in Fig. 1(b), it has been an easy task to determine the capacitances \( C_{p1}, C_{p2}, C_{s} \), and the conductances \( G_{p1}, G_{p2}, G_{s} \), via (6)-(8).

In order to apply the Galerkin method in the spectral domain to (9) and (10), it is necessary to expand \( \sigma_{ex}^{1}(\alpha, \beta) \) and \( \sigma_{ex}^{2}(\alpha, \beta) \) in a set of basis functions. In the current work, the expansion has been initially chosen in the spatial domain and then, translated into the spectral domain. The basis
functions for \( \sigma_{ex}^1(x, z) \) and \( \sigma_{ex}^2(x, z) \) have been chosen so as to reproduce the actual shape of the excess charge densities on the crossing strips as accurately as possible. This has made it possible to achieve a quick convergence of the results with respect to the number of basis functions [14].

Maxwellian distributions modulated by Chebyshev-polynomials of even order have been chosen as basis functions to approximate the variation of the excess charge densities 
functions have been more densely placed on the two regions around the mentioned regions, impredicted oscillations might take place in the piecewise approximation of the excess charge densities. Finally, it should be said that the excess charge densities have been taken to be vanishing from certain planes placed far from the crossover to the infinity (region \( |x| \geq a_1/2 \) for the strip of width \( w_1 \) and \( |z| \geq a_2/2 \) for the strip of width \( w_2 \)).

III. NUMERICAL RESULTS

In Tables I and II, we show how the excess charges on the conducting strips of two microstrip crossovers converge to a fixed value as we increase the lengths of the intervals in which the excess charge densities are assumed to be nonvanishing, i.e., \( a_1 \) and \( a_2 \). For the crossover analyzed in Table I, in which the coupling between the strips is very loose, the excess charge densities on the strips spread over a distance of width \( w_1 \) and \( |z| \leq w_1/2 \) for the strip of width \( w_2 \) in the quasi-even excitation, and over a distance of width \( w_2/2 \) for the strip of width \( w_2 \) in the quasi-odd excitation. For the crossover analyzed in Table II, in which the coupling between the strips is very tight, the excess charge densities on the strips spread over a distance of width \( w_1 \) and \( |z| \\geq w_1/2 \) in the quasi-even excitation, and over a distance of width \( w_2/2 \) in the quasi-odd excitation. These results indicate that the excess charge densities become more concentrated around the discontinuity region as the degree of coupling between the strips increases.

\[
\begin{array}{|c|c|c|c|c|}
\hline
& \frac{a_1}{h_1} = \frac{a_2}{h_1} & Q^1_{ex} (IC) & Q^2_{ex} (IC) & Q^3_{ex} (IC) \\
\hline
\text{10} & -90.7 & -93.3 & 115 & -114 \\
\text{20} & -90.8 & -92.7 & 126 & -125 \\
\text{30} & -94.3 & -96.0 & 130 & -128 \\
\text{40} & -96.1 & -97.7 & 132 & -130 \\
\text{50} & -97.1 & -98.7 & 133 & -131 \\
\text{60} & -97.8 & -99.4 & 133 & -132 \\
\text{70} & -98.2 & -99.9 & 134 & -132 \\
\text{80} & -98.6 & -100 & 134 & -133 \\
\text{90} & -98.9 & -101 & 134 & -133 \\
\text{100} & -99.1 & -101 & 134 & -133 \\
\text{150} & -98.3 & -99.8 & 132 & -131 \\
\hline
\end{array}
\]

\[
\begin{array}{|c|c|c|c|c|}
\hline
& \frac{a_1}{h_1} = \frac{a_2}{h_1} & Q^1_{ex} (IC) & Q^2_{ex} (IC) & Q^3_{ex} (IC) \\
\hline
\text{2} & -7.10 & -11.1 & 45.9 & -40.9 \\
\text{4} & -8.33 & -12.8 & 48.0 & -43.3 \\
\text{8} & -8.83 & -13.5 & 48.6 & -44.0 \\
\text{12} & -8.86 & -13.5 & 48.6 & -44.0 \\
\text{16} & -8.84 & -13.4 & 48.6 & -44.0 \\
\text{20} & -8.84 & -13.4 & 48.6 & -44.0 \\
\text{24} & -8.85 & -13.4 & - & - \\
\text{28} & -8.85 & - - & - & - \\
\text{32} & -8.86 & - - & - & - \\
\text{36} & -8.86 & - - & - & - \\
\text{40} & -8.86 & - - & - & - \\
\hline
\end{array}
\]
Fig. 2. 3-D plots of the normalized excess charge density on the lower conducting strip of a microstrip crossover. Substrate losses are neglected. (\(w_1 = w_2 = 0.1 \text{ mm}, h_1 = 0.635 \text{ mm}, h_2 = 0.01 \text{ mm}, h_3 = 10 \text{ mm}, \varepsilon_{r_1} = 9.8, \varepsilon_{r_2} = 3.4\); (a) Quasi-even excitation (\(V_1 = V_2 = 1 \text{ V}\)); (b) Quasi-odd excitation (\(V_1 = -V_2 = 1 \text{ V}\)).
the effect being more pronounced in the quasi-odd excitation than in the quasi-even excitation. In Table I, it can be seen that our results for $a_1/b_1 = a_2/b_2 \geq 70$ show very good agreement with the results presented in [9].

In Fig. 2(a) and (b), three-dimensional plots of the excess charge density on the lower strip of a microstrip crossover are shown. It can be noticed how the abrupt changes that the excess charge density experience around the crossing region of the strips (Fig. 2(b)) are reproduced in detail by using a larger number of subdomain triangular basis functions in those regions (see Section II). By comparing Fig. 2(a) and (b), it can be seen that the excess charge density in the quasi-odd excitation is more concentrated around the crossing region than the excess charge density in the quasi-even excitation. This conclusion was formerly drawn when the Tables I and II were commented.

In Fig. 3, we plot the real and imaginary part of the scattering parameters of a microstrip crossover as a function of frequency. Our results are compared with the results obtained in [10]. Good agreement is found even though the results of [10] were computed with a full-wave method of analysis and our results have been obtained by using the quasistatic equivalent circuit of the microstrip crossover described in this paper (see Fig. 1(c)) and the quasi-TEM line impedances [16].

In Fig. 4(a) and (b), our results for the capacitance parameters of the equivalent circuit of different microstrip crossovers are compared with the results obtained by other authors. In both cases, excellent agreement is found. Thus, the average discrepancies between our results and those reported in [11] lie around 2%, and the average discrepancies between our results and those reported in [6] lie around 4%.

In Fig. 5, we plot the capacitance parameters of the equivalent circuit of a crossing between two microstrip lines. The crossing takes place in a substrate composed of a plastic material and sapphire, i.e., an anisotropic dielectric material. The impedance of the line containing the lower strip is assumed to be 50 $\Omega$, whereas the impedance of the upper strip is allowed to vary between 50 $\Omega$ and 150 $\Omega$ approximately. It can be noticed that the coupling between the crossing lines increases as the impedance of the line containing the upper strip diminishes.

In Fig. 6(a) and (b), we plot the scattering parameters of a microstrip crossover on a polyimide-GaAs two-layered substrate as a function of the semiconductor loss tangent. The scattering parameters are calculated at the reference planes $T_1$ and $T_2$ of Fig. 1(b). The microstrip crossover analyzed in these two figures is of the type that appears in
sandwich type GaAs square spiral inductors and transformers where an underpass conductor track crosses the windings of the spiral [6]. In Fig. 6(a) and (b), it can be noticed that when appreciable semiconductor losses are presented, important errors may arise in the calculation of the scattering parameters of the crossover if the nonvanishing conductivity of the semiconductor is neglected. In Fig. 6(a), it can also be seen that the coupling between the crossing microstrip lines diminishes as semiconductor losses increase. Finally, it must be taken into account that the fact that the magnitude of some scattering parameters becomes bigger than one in Fig. 6(a) is not surprising since this is possible in the analysis of networks involving lossy transmission lines [18].

IV. CONCLUSION

This paper is devoted to the determination of the low-frequency equivalent circuit and the scattering parameters of the orthogonal microstrip crossover in the case in which the conducting strips of the crossover are embedded in a multilayered substrate. Isotropic and uniaxial anisotropic dielectrics as well as semiconductors with a nonnegligible conductivity are allowed to be part of the multilayered substrate. The conductances and capacitances appearing in the equivalent circuit of the microstrip crossover are obtained in terms of the complex excess charge densities on the conducting strips. These excess charge densities are numerically computed by using the Galerkin method in the spectral domain. The results obtained are compared with results formerly published by other authors, and good agreement is found. Original results are presented for the equivalent circuit parameters and the scattering of microstrip crossover printed on substrates involving anisotropic dielectrics and doped semiconductors with a nonnegligible conductivity.

APPENDIX

In this appendix, recurrence algorithms are provided to calculate the spectral functions \( G(\alpha, y = H_N, y' = H_{N'}, \beta) \), which appear in the expressions (9) and (10) of the paper. These spectral functions stand for the two-dimensional Fourier transforms of the Green's function of the multilayered substrate shown in Fig. 1(a) when the source and field points are alternatively placed at the interfaces \( y = H_N \) and \( y = H_{N'} \) of the substrate. Before showing how to calculate \( G(\alpha, y = H_N, y' = H_{N'}, \beta), (i, j = 1, 2) \), we are going to define some some spectral functions which depend on the elements of the permittivity tensors, \( \epsilon_i, (i = 1, \ldots, N_3) \), and the thickness \( h_i \) (\( i = 1, \ldots, N_3 \)), of the layers of the substrate (see (1) and Fig. 1(a)). These spectral functions are given by:

\[
\delta_i(\alpha, \beta) = \left[ \delta_i(\alpha, \beta) \text{coth} \left( \frac{\delta_i(\alpha, \beta) h_i}{p_{y,i}} \right) + \delta_{i+1}(\alpha, \beta) \text{coth} \left( \frac{\delta_{i+1}(\alpha, \beta) h_{i+1}}{p_{y,i+1}} \right) \right]
\]

\[
\hat{g}_{i,i-1}(\alpha, \beta) = -\hat{g}_i(\alpha, \beta) \text{coth} \left( \frac{\hat{q}_i(\alpha, \beta) h_i}{p_{y,i}} \right)
\]
where

$$\tilde{q}_{i}(\alpha, \beta) = \left[ \frac{p_{x,i}p_{y,i}(\alpha^2 + \beta^2)}{2\pi f_{c0}} \right]^{1/2}$$

(12) By using the reciprocity theorem and the fact that the spectral functions defined in (11) and (12) are even functions of \(\alpha\) and \(\beta\), it can be easily shown that

$$H_{21}(\alpha, \beta) = H_{12}(-\alpha, -\beta) = H_{12}(\alpha, \beta)$$

(31)

In order to construct the recurrent algorithms written in (21)–(30), which are the bases for calculating the matrices \(\overline{H}\) and \(\overline{G}\) defined in (16) and (17), the ideas exposed in [19] have been used. The recurrent algorithms are very well adapted to being implemented in a computer code, owing to their numerical stability.

REFERENCES


Jesus Martel was born in Seville, Spain, in December 1966. He received the B.Sc. and M.Sc. degrees in physics from the University of Seville, Seville, Spain in 1989 and 1990, respectively. He is currently pursuing the Ph.D. at the same university. His research interests are in the modeling of planar line discontinuities.

Rafael R. Boix was born in Melilla, Spain, in 1962. He received the B.Sc., M.Sc., and Ph.D. degrees in physics, all from the University of Seville, Seville, Spain, in 1985, 1986, and 1990, respectively. Since 1990, he has been an Assistant Professor at the University of Seville. His research interests are in the modeling of planar line discontinuities and printed-circuit antennas.

Manuel Horno (M'75) was born in Torre del Campo, Jaén, Spain. He received the B.Sc. and the Ph.D. degrees in physics from the University of Seville, Seville, Spain, in 1969 and 1972, respectively. Since October 1969, he has been with the Department of Electricity and Electronics at the University of Seville, where he became an Assistant Professor in 1970, and Associate Professor in 1975, and Professor in 1986. His research interests include boundary value problems in electromagnetic theory, wave propagation through anisotropic media and microwave integrated circuits. He is presently engaged in the analysis of planar transmission lines embedded in anisotropic materials, multiconductor transmission lines, microstrip discontinuities and planar slow-wave structures.