Resonant Modes of Circular Microstrip Patches
Over Ground Planes with Circular Apertures
in Multilayered Substrates Containing
Anisotropic and Ferrite Materials

Vicente Losada, Rafael R. Boix, Member, IEEE, and Manuel Horno, Member, IEEE

Abstract—In this paper, the authors analyze how the resonant modes of circular microstrip patch resonators are affected by the presence of circular apertures in the ground plane located under the patches. A rigorous full-wave analysis in the Hankel transform domain (HTD) is carried out in order to obtain the resonant frequencies, quality factors, and radiation patterns of the circular microstrip patch resonators over ground planes with circular apertures. With the use of suitable Green’s functions in the HTD, the analysis is performed for the case where the circular patches, as well as the ground planes containing the apertures are embedded in a multilayered substrate consisting of isotropic dielectrics, uniaxial anisotropic dielectrics, and/or magnetized ferrites. The numerical results obtained are compared with experimental results, and good agreement is found. The results show that the circular apertures significantly affect the resonant frequencies of circular microstrip patches.

Index Terms—Microstrip patches with tuning slots, microwave planar circuits, resonators.

I. INTRODUCTION

CIRCULAR microstrip patch resonators can be used either as antennas or as components of oscillators and filters in microwave integrated circuits [1]. Since the bandwidth of microstrip patch resonators around their operating resonant frequencies is very narrow [1], it is important to develop accurate algorithms for the prediction of those resonant frequencies.

When a microstrip patch resonator acts as an antenna, it can be fed by a microstrip line located below the ground plane of the antenna through an aperture in the ground plane [2], [3]. This feeding configuration has been found very advantageous for several reasons. For instance, it makes it possible to use a high dielectric-constant substrate for the feeding network and a low dielectric-constant substrate for the antenna element, which yields optimal performance for both the feeding network and antenna element [3]. Also, the radiation arising from the feeding network cannot interfere with the main radiation pattern generated by the antenna since the ground plane separates the two radiation mechanisms [2]. It should be pointed out that the presence of apertures in the ground plane of microstrip patch antennas unavoidably affects the resonant properties of the antennas. This effect of ground-plane apertures on microstrip patches has been explicitly shown in [4] and [5], where the authors have demonstrated that apertures in the ground plane of rectangular microstrip patches can be used as a way to tune their resonant frequencies. Since ground-plane apertures can play a role in the design of microstrip patch antennas and microstrip patch circuit components, the computer codes developed for the analysis of microstrip patch resonators should be able to account for the effect of possible apertures existing in the ground plane of the resonators.

Setting aside the topic of microstrip patches over ground planes with apertures, in the last few years, there is some interest on studying how the performance of microstrip patches is affected when anisotropic dielectrics and/or magnetized ferrites are used as substrates. Bearing in mind that some of the standard materials used as substrates of printed circuit and antennas exhibit dielectric anisotropy [6], Pozar has shown that substrate dielectric anisotropy should be taken into account when designing microstrip patch antennas because if anisotropy were ignored, the antennas might operate out of the expected frequency band [7]. Aside from that, magnetized ferrites have proven to have potential application as substrates of microstrip patch antennas. For instance, measurements have shown that the resonant frequencies of microstrip antennas printed on ferrite substrates can be varied over a wide frequency range by adjusting the bias magnetic field [8]. Also, ferrite substrates can be used for reducing the radar cross section of microstrip antennas [9] and for achieving circularly polarized microstrip antennas with a single feed [9].

In this paper, the authors analyze how the resonant modes of circular microstrip patch resonators are affected by the presence of circular apertures in the ground planes existing underneath the patches. The Galerkin’s method in the Hankel transform domain (HTD) [10] is applied to the full-wave computation of the resonant frequencies, quality factors, and radiation patterns of the resonant modes of the circular microstrip patches over ground planes with circular apertures. The metallic patches, as
well as the apertures, are embedded in a multilayered media, which may contain uniaxial anisotropic dielectrics and/or magnetized ferrites. As far as the authors know, the only published results on the full-wave analysis of microstrip patch resonators over ground planes with apertures refer to patches and apertures of rectangular geometry on isotropic dielectric substrates [4], [5]. Therefore, in this paper, focus is on results for patches and apertures of circular geometry on both isotropic and anisotropic substrates. In Section II, the authors provide details of the application of the Galerkin's method in the HTD to the analysis of circular microstrip resonators over ground planes with circular apertures. In Section III, numerical results are presented for a variety of circular microstrip resonators over circular apertures fabricated on different types of substrates. The numerical results obtained for resonators on isotropic dielectric substrates are compared with experimental results, and good agreement is found.

II. OUTLINE OF THE NUMERICAL PROCEDURE

Fig. 1 shows the side and top views of a circular microstrip patch of radius $a$ over a ground plane with a circular aperture of radius $b$. The rotation axis of the aperture is assumed to be coincident with that of the patch ($z$-axis in Fig. 1). Both the circular metallic patch and ground plane are assumed to be perfect electric conductors (PECs) of neglecting thickness. Also, the patch and ground plane are embedded in a three-layered substrate (although the method described in this section can be easily extended to deal with multilayered substrates containing any arbitrary number of layers). The time variation of the electromagnetic fields in the structure of Fig. 1 is assumed to be specified by a $e^{-j\omega t}$ factor (where $\omega$ is complex to account for radiation losses), which will be suppressed throughout. The layers of the substrate are assumed to be of infinite extent along the $x$- and $y$-coordinates, and these layers are also assumed to be made of any of the three following materials: lossless isotropic dielectrics, lossless uniaxial anisotropic dielectrics, and lossless magnetized ferrites with applied bias magnetic field. In case the $i$th layer of the substrate ($i = 1, 2, 3$) is an isotropic dielectric, its permittivity will be taken as $\varepsilon_{0}\varepsilon_r^{i}$ and its permeability will be taken as $\mu_0$. In case the $i$th layer ($i = 1, 2, 3$) is an uniaxial anisotropic dielectric, it will be assumed that its optical axis is the $z$-axis [6]. For that case, the permeability of the anisotropic dielectric will be taken as $\mu_0$ and the permittivity tensor will be taken as [7]

$$\varepsilon_i = \varepsilon_0 \begin{pmatrix} \varepsilon_r^i & 0 & 0 \\ 0 & \varepsilon_r^i & 0 \\ 0 & 0 & \varepsilon_r^i \end{pmatrix}.$$ (1)

In case the $i$th layer of the substrate ($i = 1, 2, 3$) is a magnetized ferrite, it will be assumed that the bias magnetic field of the ferrite is directed along the $z$-axis. For that case, the permeability of the ferrite material will be taken as $\mu_0\mu_f^i$ ($i = 1, 2, 3$) and the permeability tensor will be taken as [9]

$$\mu_i = \mu_0 \begin{pmatrix} \mu_r^i & -j\mu_s^i & 0 \\ j\mu_s^i & \mu_r^i & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$ (2)

Throughout this paper, ferrite materials will be assumed to be magnetically saturated. Under this assumption, the elements of the permeability tensor of (2), i.e., $\mu_i$, can be obtained in terms of the gyromagnetic ratio $\gamma = 1.759 \times 10^{14}$ C/Kg, the saturation magnetization of the ferrite material $M_s$, and the internal bias magnetic field $H_B$, as explained in [11].

Let $\mathbf{j}_1(\rho, \phi) = j_{1x}(\rho, \phi)\hat{\rho} + j_{1y}(\rho, \phi)\hat{\phi}$ be the surface current density on the ground plane with a circular aperture and let $\mathbf{j}_2(\rho, \phi) = j_{2x}(\rho, \phi)\hat{\rho} + j_{2y}(\rho, \phi)\hat{\phi}$ be the surface current density on the circular patch when cylindrical coordinates are used ($x = \rho \cos \phi$ and $y = \rho \sin \phi$). Also, let $\mathbf{E}_1(\rho, \phi, z) = E_{1x}(\rho, \phi, z)\hat{\rho} + E_{1y}(\rho, \phi, z)\hat{\phi}$ and $\mathbf{E}_2(\rho, \phi, z) = E_{2x}(\rho, \phi, z)\hat{\rho} + E_{2y}(\rho, \phi, z)\hat{\phi}$ be the values of the transverse electric field (due to $\mathbf{j}_1(\rho, \phi)$ and $\mathbf{j}_2(\rho, \phi)$) at the plane of the aperture and at the plane of the patch respectively (see Fig. 1). Owing to the revolution symmetry of the multilayered medium of Fig. 1 around the $z$-axis, when the Helmholtz equations for the axial field components $E_z$ and $H_z$ are solved in cylindrical coordinates inside each of the layers of that medium, it turns out that the dependence of $E_z$ and $H_z$ on the $\phi$ coordinate is of the type $e^{j\beta z}$ ($m = \ldots, -2, -1, 0, 1, 2, \ldots$) and, as a consequence, $\mathbf{j}_1(\rho, \phi)$ and $\mathbf{E}_1(\rho, \phi, z = d_i)$ ($i = 1, 2$) can be written as

$$\mathbf{j}_1(\rho, \phi) = (d_{i1}(\rho, \phi)\hat{\rho} + d_{i2}(\rho, \phi)\hat{\phi}) e^{jm\phi}, \quad i = 1, 2;$$

$m = \ldots, -2, -1, 0, 1, 2, \ldots$ (3)

$$\mathbf{E}_1(\rho, \phi, z = d_i) = E_{1m}(\rho, \phi)\hat{\rho} + E_{1n}(\rho, \phi)\hat{\phi} e^{jm\phi}, \quad i = 1, 2;$$

$m = \ldots, -2, -1, 0, 1, 2, \ldots$ (4)

By virtue of (3) and (4), it is possible to follow a mathematical reasoning very similar to that shown in [12, eq. (1)-(14)] for...
obtaining a relation among \( j_1(\rho, \phi), j_2(\rho, \phi), E_0(\rho, \phi, z = d_1), \)
and \( E_0(\rho, \phi, z = d_2) \) in the spectral domain given by
\[
\begin{align*}
\mathbf{\tilde{H}}^H_{1m}(k_{p_0}) &= \mathbf{\hat{H}}^H_{1m}(k_{p_0}) \cdot \mathbf{\hat{E}}^H_{1m}(k_{p_0}) + \mathbf{\hat{H}}^H_{2m}(k_{p_0}) \cdot \mathbf{\hat{E}}^H_{2m}(k_{p_0}) \\
\mathbf{\tilde{E}}^H_{m}(k_{p_0}) &= \mathbf{\hat{H}}^H_{m}(k_{p_0}) \cdot \mathbf{\hat{E}}^H_{m}(k_{p_0}) + \mathbf{\hat{H}}^H_{m}(k_{p_0}) \cdot \mathbf{\hat{E}}^H_{m}(k_{p_0})
\end{align*}
\]
(5)
(6)
where \( \mathbf{\tilde{H}}^H_{1m}(k_{p_0}) \) and \( \mathbf{\tilde{E}}^H_{m}(k_{p_0}) \) \((i = 1, 2)\) are vector functions that have to be determined in terms of the \((m + 1)\)th and \((-m - 1)\)th Hankel transforms of \( j_{i+1}(\rho, \phi); j_{i+1}(\rho, \phi); E_{0i}(\rho, \phi); E_{0i}(\rho, \phi); E_{0i}(\phi, \rho); \) and \( E_{0i}(\phi, \rho); \) \((i = 1, 2)\) as follows:
\[
\begin{align*}
\mathbf{\tilde{H}}^H_{m}(k_{p_0}) &= \int_{0}^{\infty} \left[ \begin{array}{c}
j_{i+1}(\rho, \phi) \\
j_{i+1}(\rho, \phi)
\end{array} \right] J_{m+1}(k_{p_0}) \rho d\rho, \\
i = 1, 2; m = \ldots, -2, -1, 0, 1, 2, \ldots
\end{align*}
\]
(7)
Equations (5) and (6) stand for the HTD equivalent of [5, eq. (5)]. The four \( 2 \times 2 \) matrices \( \mathbf{\hat{H}}^H_{ij}(k_{p_0}) \) \((i, j = 1, 2)\) stand for a set of dyadic Green’s functions in the HTD. As explained in [12] (see (9) and (14)), all these dyadic Green’s functions can be obtained in a straightforward way in terms of their two-dimensional Fourier transform (TDFT) version, which, in turn, can be determined very efficiently by using an algorithm based on the equivalent boundary method [13].

Once the algorithm for the computation of the matrices \( \mathbf{\hat{H}}^H_{ij}(k_{p_0}) \) \((i, j = 1, 2)\) is available, the Galerkin’s method in the HTD can be applied to (5) and (6) for obtaining the resonant frequencies and quality factors of the resonant modes of the structure of Fig. 1. When applying the Galerkin’s method, both the transverse electric field on the aperture and the current density on the patch have to be approximated by known basis functions. After following a procedure analogous to that described in [10], a nonlinear eigenvalue equation for the complex angular frequency \( \omega \) is obtained. If we let \( \omega_{mn} = 2\pi(f_{mn} + jf_{mn}^*) \) \((m = \ldots, -2, -1, 0, 1, 2, \ldots; n = 1, 2, 3, \ldots)\) be the set of complex roots of the aforementioned nonlinear equation, the quantities \( f_{mn} \) \((m = \ldots, -2, -1, 0, 1, 2, \ldots; n = 1, 2, 3, \ldots)\) stand for the resonant frequencies of the circular microstrip patch of Fig. 1 and the quantities \( Q_{mn} = f_{mn}^*/2f_{mn}^* \) \((m = \ldots, -2, -1, 0, 1, 2, \ldots; n = 1, 2, 3, \ldots)\) stand for the quality factors [1].

Once \( f_{mn} \) and \( Q_{mn} \) are known for a particular resonant mode of the structure of Fig. 1, one can derive expressions for the vector functions \( \mathbf{\tilde{H}}^H_{1m}(k_{p_0}) \) and \( \mathbf{\tilde{E}}^H_{m}(k_{p_0}) \) \((i = 1, 2)\) corresponding to that particular resonant mode [see (5) and (6)]. All these functions can be used in conjunction with the equivalent boundary method [13] for deriving the TDFT of the transverse electric field at the upper and lower limiting planes of the multilayered structure in Fig. 1. The expressions of these two TDFT can then be used for computing both the radiation electric field \( \mathbf{E}(r', \theta', \phi') \) in the air upper half-space of Fig. 1 and the radiation electric field \( \mathbf{E}(r'', \theta'', \phi'') \) in the air lower half-space \( \{r', \theta', \phi'\} \) and \( \{r'', \theta'', \phi''\} \) are local sets of spherical coordinates defined in Fig. 1) by means of the stationary phase method (see [14, pp. 164–169]).

The basis functions chosen in this paper for approximating both the transverse electric field on the circular aperture and the current density on the circular patch have different expressions for axial-symmetrical resonant modes \((m = 0)\) and for nonaxial symmetrical resonant modes \((m \neq 0)\). In the case of axial-symmetrical modes, the basis functions chosen for \( E_{0i}(\rho, \phi); E_{0i}(\rho, \phi); E_{0i}(\phi, \rho); \) and \( E_{0i}(\phi, \rho) \) [see (3) and (4)] are given by
\[
\begin{align*}
E_{0i}(\rho, \phi) &= \frac{T_{2i-2}(\rho/b)}{\sqrt{1-(\rho/b)^2}}(\rho/b), \quad i = 1, \ldots, N_1 + 1 \\
E_{0i}(\rho, \phi) &= U_{2i-1}(\rho/b)\sqrt{1-(\rho/b)^2}, \quad i = 1, \ldots, N_1 \\
E_{0i}(\rho, \phi) &= U_{2i-1}(\rho/a)\sqrt{1-(\rho/a)^2}, \quad i = 1, \ldots, N_2 \\
E_{0i}(\rho, \phi) &= \frac{T_{2i-2}(\rho/a)}{\sqrt{1-(\rho/a)^2}}(\rho/a), \quad i = 1, \ldots, N_2 + 1
\end{align*}
\]
(9)
(10)
(11)
(12)
where \( U_{2i-1}(\rho) \) and \( T_{2i-2}(\rho) \) stand for the Chebyshev polynomials of the second and first kinds, respectively.

For nonaxial symmetrical modes, the basis functions chosen for approximating \( E_{0i}(\rho, \phi); E_{0i}(\rho, \phi); j_{2i}(\rho, \phi); \) and \( j_{2i}(\rho, \phi) \) when \( m \neq 0 \) [see (3) and (4)] are given by
\[
\begin{align*}
E_{0i}(\rho, \phi) &= \frac{T_{2i-2}(\rho/b)}{\sqrt{1-(\rho/b)^2}}(\rho/b)^{m-1}, \\
m = \ldots, -2, -1, 1, 2, \ldots; \quad i = 1, \ldots, N_1 + 1 \\
E_{0i}(\rho, \phi) &= U_{2i-1}(\rho/b)\sqrt{1-(\rho/b)^2}(\rho/b)^{m-2}, \\
m = \ldots, -2, -1, 1, 2, \ldots; \quad i = 1, \ldots, N_1 \\
j_{2i}(\rho, \phi) &= U_{2i-1}(\rho/a)\sqrt{1-(\rho/a)^2}(\rho/a)^{m-2}, \\
m = \ldots, -2, -1, 1, 2, \ldots; \quad i = 1, \ldots, N_2 \\
j_{2i}(\rho, \phi) &= \frac{T_{2i-2}(\rho/a)}{\sqrt{1-(\rho/a)^2}}(\rho/a)^{m-1}, \\
m = \ldots, -2, -1, 1, 2, \ldots; \quad i = 1, \ldots, N_2 + 1.
\end{align*}
\]
(13)
(14)
(15)
(16)
The basis functions used for the approximation of the current density on the circular patch of Fig. 1 [see (11), (12), (15), and (16)] are very similar to those used in [12] for approximating the current density on a circular patch over a ground plane without aperture. Also, the same basis functions have been used for approximating the magnetic current density on the aperture \( \mathbf{M}_1(\rho, \phi) = -\hat{Z} \times \mathbf{E}_0(\rho, \phi, z = d_1) \) [see (9), (10), (13), and (14)] in accordance with the concept of complementary electromagnetic structures [15]. In [12], the authors demonstrated that the basis functions of the type shown in (9)–(16) are very appropriate for the HTD analysis of microstrip patches over ground planes without apertures in multilayered isotropic dielectric
substrates for three reasons: they ensure a quick convergence of the Galerkin’s method in the HTD with respect to the number of basis functions, their Hankel transforms can be obtained in closed form in terms of spherical Bessel functions (see [12, Appendix B]), and they lead to HTD infinite integrals, which are amenable to asymptotic analytical integration techniques (see [12, Sec. IV, Appendix C]). Fortunately, the authors have found that these advantages are all kept when the same basis functions are used in the HTD analysis of circular patches over ground planes with apertures in multilayered substrates containing anisotropic and ferrite materials.

III. NUMERICAL AND EXPERIMENTAL RESULTS

Fig. 2 presents numerical and experimental results for the resonant frequencies of the first three resonant modes of circular microstrip patches over ground planes with and without circular apertures. It is noticed that whereas for the modes \( m = 1, n = 1 \) and \( m = 2, n = 1 \), the resonant frequencies of the patches over ground planes without apertures are larger than those obtained with apertures, the opposite occurs for the mode \( m = 0, n = 1 \). In fact, the results obtained for the resonant frequencies of the mode \( m = 1, n = 1 \) when \( b = 0 \) lie around 15% above those obtained when \( b = 0.5a \) and around 23% above those obtained when \( b = a \). For the mode \( m = 2, n = 1 \), the results obtained for the resonant frequencies when \( b = 0 \) lie around 5% above those obtained when \( b = 0.5a \) and around 13% above those obtained when \( b = a \). Finally, for the mode \( m = 0, n = 1 \), the results obtained for the resonant frequencies when \( b = 0 \) lie around 50% below those obtained when both \( b = 0 \) and \( b = a \). In the experiments, the resonators have been fed by open-ended microstrip lines capacitively coupled to the circular metallic patches and the resonant frequencies have been calculated using closed forms in terms of spherical Bessel functions (see [12, Sec. IV, Appendix C]).

FIG. 2. Resonant frequencies of circular microstrip patches printed on an isotropic dielectric (250-LX-0193-43-11: \( h_2 = 0.49 \) mm, \( r_c^2 = 2.43 \)) over ground planes with and without circular apertures. Numerical results (solid, dashed, and dotted lines) are compared with experiments (*).
conducting plane below the ground plane containing the aperture. Numerical experiments have shown that, in the cases analyzed in Fig. 4, this additional conducting plane can be placed at a distance of the ground planes containing the apertures, which is only five times the thickness of the dielectric substrate without substantially affecting the resonant properties of the microstrip patch.

In Fig. 5, results are presented for the resonant frequencies of the fundamental mode of a circular microstrip patch over ground planes with and without circular apertures. The microstrip patch is assumed to be covered by a dielectric superstrate. Dielectric superstrates of this type (radomes) are placed on top of microstrip patch antennas for protecting the antennas against environmental hazards, such as rain, fog, and snow [16]. Fig. 5 shows that the presence of the superstrate not only affects the resonant frequency of the fundamental mode of the microstrip patch over ground plane without aperture, but it also affects the resonant frequencies of the fundamental mode of the microstrip patch with apertures. Thus, as $h_3$ changes from zero to $3h_2$, $f_1^{11}$ decreases 8% when $b = 0$, $f_1^{11}$ decreases 8% when $b = 0.5a$, and $f_1^{11}$ decreases 12% when $b = a$. It turns out that the resonant frequencies of the structures with superstrate are always smaller than those without superstrate because the effective permittivity of the medium surrounding the patch is higher in the former case than in the latter case.

In Fig. 6, the numerical and experimental results obtained for the resonant frequencies of the first three resonant modes of circular microstrip patches over ground planes without apertures are compared with the results obtained for the same microstrip patches over ground planes with apertures in two different situations. In the first situation, the air occupies the space below the apertures and, in the second situation, there is a dielectric layer of high permittivity below the apertures. This latter situation has an interest for antenna applications since when microstrip patch antennas are fed by microstrip lines through apertures, the feeding microstrip lines may be printed on high-permittivity dielectric substrates [2], [3]. It can be noticed that the results obtained for the resonant frequencies of the patches with apertures in contact with a high-permittivity dielectric layer not only substantially differ from the results without apertures, but they also appreciably differ from the results obtained with apertures in contact with air. Thus, in the case of the mode $m = 1, n = 1$, the results obtained for the resonant frequencies when $b = a$ and $\epsilon_r^2 = 10$ are around 22% below those obtained when $b = a$ and $\epsilon_r^2 = 1$. In the case of the mode $m = 2, n = 1$, the results obtained for the resonant frequencies when $b = a$ and $\epsilon_r^4 = 10$ are around 23% below those obtained when $b = a$ and $\epsilon_r^4 = 1$. Finally, in the case of the mode $m = 0, n = 1$, the results obtained for the resonant frequencies when $b = a$ and $\epsilon_r^4 = 10$
are, on average, 10% below those obtained when \( b = a \) and \( e_3^2 = 1 \). This clearly indicates that the placement of a dielectric layer of high permittivity below the ground plane of a circular microstrip patch over ground plane with aperture may considerably change the resonant frequencies of the patch. Concerning the degree of discrepancy between the numerical and experimental results shown in Fig. 6, it should be said that, whereas the differences between numerical and experimental results are within 4% when \( b = a \) and \( e_3^2 = 1 \), these differences are within 2% when \( b = a \) and \( e_3^2 = 10 \).

In Fig. 7, results are presented for the resonant frequencies of the fundamental mode of circular microstrip patches over ground planes with and without circular apertures in the case where the patches are printed on an anisotropic dielectric substrate, i.e., Epsilam-10. In this figure, the results obtained for the patches printed on an anisotropic Epsilam-10 are compared with the results that would be obtained if the anisotropy of Epsilam-10 were neglected. When \( b = 0 \) and \( b = 0.5a \), the differences between the results obtained considering anisotropy and neglecting anisotropy slowly diminish from 3% to 1% as \( a/h_2 \) varies in the range from 2 to 10. However, in the cases where \( b = a \), these differences remain fixed around 5% regardless of the variation of \( a/h_2 \). Therefore, the effect of dielectric anisotropy is much more pronounced in the cases where \( b = a \) than in the cases where \( b = 0 \) and \( b = 0.5a \). This is attributed to the fact that when \( b = a \), the fringing electric field at the edge of the patches is more appreciable than that existing when \( b = 0 \) and \( b = 0.5a \) and, therefore, when \( b = a \) there is a larger portion of electric field that sees the permittivity in the directions parallel to the plane of the patch—\( e_2^2 \)—and a smaller portion of electric field that sees the permittivity in the direction perpendicular to the plane of the patch—\( e_3^2 \).

Fig. 8 plots the results for the resonant frequencies of the first two resonant modes \((m = 1, n = 1) \) and \((m = -1, n = 1) \) of a circular microstrip patch over ground planes with and without circular apertures when the substrate of the patch is a magnetized ferrite. In this figure, the resonant frequencies are plotted versus the magnitude of the bias magnetic field. It can be noticed that these resonant frequencies can be tuned over a wide frequency range by the bias magnetic field not only when the patch is placed over a ground plane without aperture [8], [9], but also when the patch is placed over ground planes with apertures. In fact, when \( \mu_0 H_0^2 \) is varied from 0 to 0.3 T, the resonant frequencies of the resonant mode \( m = -1, n = 1 \) change more than 50% in all the cases treated in Fig. 8 (when \( b = 0 \), \( b = 0.4a \), and \( b = 0.8a \) above the magnetostatic mode region, as well as below the magnetostatic mode region), and the resonant frequencies of the resonant mode \( m = 1, n = 1 \) change more than 25%. As in [17], the authors have found a cutoff frequency region \( \mu_0 H_0^2/2\pi < \mu_0\gamma\sqrt{(H_0^2 + H_0^2 + M_2^2)/2\pi} \) in which resonances are not allowed due to the propagation of an infinite number of magnetostatic volume-wave modes along the conductor-backed ferrite layer supporting the circular patch [17]. As a result, the resonant frequencies of all resonant modes appear below the cutoff region and above the cutoff region [17]. Fig. 8 shows that, for every value of \( \mu_0 H_0^2 \), the differences between the resonant frequencies of the circular patch with and without apertures are much more pronounced in the case of the resonant mode \( m = -1, n = 1 \) than in the case of the mode \( m = 1, n = 1 \) above the magnetostatic mode region. However, the same differences are more pronounced in the case of the resonant mode \( m = 1, n = 1 \) than in the case of the mode \( m = -1, n = 1 \) below the magnetostatic mode region.

IV. CONCLUSIONS

The Galerkin’s method in the HTD has been used for the numerical calculation of the resonant frequencies, quality factors, and radiation patterns of the first resonant modes of circular microstrip patch resonators over ground planes with circular apertures in the case in which the patches as well as the ground planes containing the apertures are embedded in a multilayered substrate that may contain uniaxial anisotropic dielectrics and/or magnetized ferrites. In order to test the validity of the
analysis, numerical results obtained for the resonant frequencies have been compared with measurements, and good agreement has been found. The results show that the resonant frequencies of circular microstrip patches over ground planes with circular apertures can be significantly different from the resonant frequencies of the patches in the absence of apertures, especially when the size of the apertures is comparable to that of the patches and/or there is a high-permittivity dielectric layer below the apertures. Also, the effect of the apertures has been found to be more pronounced in axial-symmetrical resonant modes than in nonaxial-symmetrical resonant modes. Dielectric anisotropy effects have proven to be especially significant when the size of the apertures is similar to that of the patches. Finally, numerical results obtained for circular patches with apertures printed on ferrite substrates have shown that the resonant frequencies of these patches can be tuned by means of the bias magnetic field.

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