Abstracts

Sergei Artemov

Reflection vs. co-Reflection in Intuitionistic Epistemic Logic.

This is joint work with Tudor Protopopescu. We outline an intuitionistic view of knowledge which maintains the Brouwer-Heyting-Kolmogorov semantics and is consistent with Williamson’s suggestion that intuitionistic knowledge is the result of verification. On this view, co-Reflection $F \to KF$ is valid since it encodes a fundamental property that intuitionistic truth is constructive and hence checkable. On the other hand, reflection $KF \to F$ demands that all verifications yield strict proofs which is not consistent with verification practice and hence is not among the basic principles of intuitionistic epistemic logic. Consequently we show that reflection of knowledge is a distinctly classical principle, too strong as the intuitionistic truth condition for knowledge, “false is not known,” which can be more adequately expressed by e.g., $\neg(KF \& \neg F)$, or, equivalently, $\neg K \bot$.

We construct a system of intuitionistic epistemic logic

$$IEL = IPC + K(F \to G) \to (KF \to KG) + F \to KF + \neg K \bot$$

and provide its provability/verification semantics by extending the well-known Gödel’s embedding. We also show that IEL enjoys a natural and explanatory Kripke-style semantics.

Lev Beklemishev

Positive modal logic and reflection principles

TBA
This talk reports on joint work with Albert Visser. Suppose we start with a foundational base theory $B$ formulated in a language $L$ (such as $B =$ Peano arithmetic, or $B =$ Zermelo-Fraenkel theory), and we extend $B$ to a new theory $B^+ := B \cup \Sigma$, where $\Sigma$ is a set of sentences formulated in the language $L^+ := L \cup \{S\}$ that describes certain features of a Tarskian satisfaction predicate $S$.

I will discuss the status of our current knowledge of the relationship between $B$ and $B^+$ in connection with the following four questions for various choices of $B$ and $\Sigma$:

1. Is $B^+$ semantically conservative over $B$? In other words, does every model of $B$ expand to a model of $B^+$?
2. Is $B^+$ syntactically conservative over $B$? In other words, if some $L$-sentence $\phi$ is provable from $B^+$, then is $\phi$ also provable from $B$?
3. Is $B^+$ interpretable in $B$?
4. What type of speed-up (if any) does $B^+$ have over $B$?

A recent approach by Beklemishev uses provability logics to represent reflection principles in formal theories and uses said principles to calibrate a theory’s consistency strength. There are several benefits to this approach, including semi-finitary consistency proofs and independent combinatorial statements.

A key ingredient is Japaridze’s polymodal provability logic $\text{GLP}_\omega$. In order to study stronger theories one needs to go beyond $\text{GLP}_\omega$ to the logics $\text{GLP}_\Lambda$, where $\Lambda$ is an arbitrary ordinal. These logics have for each ordinal $\xi < \Lambda$ a modality $\langle \xi \rangle$. Proof theoretic ordinals below $\Gamma_0$ may be represented in the closed fragment of $\text{GLP}_\Lambda$ worms therein. Worms are iterated consistency statements of the form $\langle \xi_n \rangle \cdots \langle \xi_1 \rangle \top$ and are well-ordered by their consistency strength.

We present a calculus for computing the order types of worms and compare the resulting ordinal representation system with standard systems based on Veblen functions. We will also discuss how larger ordinals arising from impredicative proof theory may be represented within provability logics.
Melvin Fitting
Realization, with an implementation

Gödel inaugurated a project of finding an arithmetic semantics for intuitionistic logic, but did not complete it. It was finished by Sergei Artemov, in the 1990s. As part of this work, Artemov introduced the first justification logic, LP, (standing for logic of proofs). This is a modal-like logic, with an infinite family of proof or justification terms, and can be seen as an explicit version of the well-known modal logic S4. Since then, many other justification logic/modal logic pairs have been investigated, and justification logic has become a subject of independent interest, going beyond the original connection with intuitionistic logic. It is now known that there are infinitely many justification logics, but the exact extent of the family is not known. Justification logics are connected with their corresponding modal logics via a Realization Theorem. A Realization Theorem connecting LP and S4 has a constructive proof, but there are other cases for which realization holds, but it is not known if a constructive proof exists. I will discuss Realization Theorems in general, and LP/S4 in particular. The realization proof I will talk about has a two part structure, going through a quasi-realization stage. This gives some additional insight into the phenomenon. I will conclude by demonstrating a computer implementation of the algorithm behind realization for LP.

Kentaro Fujimoto
Considering “Turing-Feferman" type progressions for set theory.

I’ll consider several types of Turing-Feferman type progression particularly for set theory, which involve logical or set-theoretical principles to which we “implicitly commit” in accepting set theory.

Eduardo Hermo-Reyes
A modal logic for transfinite Turing progressions

It is known that the modal logic GLP can be used to directly denote finite levels at Turing Progressions. However, at transfinite levels, this link between GLP and Turing progressions is only by approximating them. Using a new modal signature, we present a logic that generates valid principles that hold between the different Turing progressions and that can be used to directly denote these transfinite levels.

Joost Joosten
Turing-Taylor expansions for arithmetical theories

Turing progressions are obtained by (transfinitely) iterating adding consistency statements to some base theory. Different consistency statements yield different progressions. We see how we can use these Turing progressions to describe well-known arithmetical theories. Just like analytic functions can be expressed as a Taylor expansion being a sum of monomials, we see that various theories admit expressing them as unions of Turing progressions. Some preliminary results are presented in http://arxiv.org/abs/1404.4483
Robert Lubarsky
*Separating Variants of LEM, LPO, and MP*

We separate most of the basic fragments of classical logic which are used in reverse constructive mathematics. A group of related topological models are used to show that WLEM does not imply LPO (and thus WMP – Weak Markov’s Principle – is not provable in IZF) and to separate Richman’s \(LLPO_n\) hierarchy. A simple Kripke model is used to separate WLPO and WKL, and some topological models separate WMP from MP.

Elena Nogina
*On a Hierarchy of Reflection Principles in Peano Arithmetic*

We study reflection principles of Peano Arithmetic \(PA\) based on both proof and provability predicates. Let \(P\) be a propositional letter and each of \(Q_1, Q_2, \ldots, Q_m\) is either ‘\(\Box\)’ standing for provability in \(PA\), or ‘\(\dot{u}\)’ standing for ‘\(\dot{u}\) is a proof of . . . in \(PA\)’, \(u\) is a fresh proof variable. Then the formula \(Q_1Q_2\ldots Q_mP \rightarrow P\) is called generator, and the set of all its arithmetical instances is the reflection principle corresponding to this generator. We will refer to reflection principles using their generators. It is immediate that all reflection principles without explicit proofs (\(Q_i = \Box\) for all \(i\)) are equivalent to the local reflection principle \(\Box P \rightarrow P\). All \(\Box\)-free reflection principles are provable in \(PA\) and hence equivalent to \(\dot{u} : P \rightarrow P\). Mixing explicit proofs and provability yields infinitely many new reflection principles.

**Theorem 1.** Any reflection principle in \(PA\) is equivalent to either \(\Box P \rightarrow P\) or \(\Box^k\dot{u} : P \rightarrow P\) for some \(k \geq 0\).

**Theorem 2.** Reflection principles constitute a non-collapsing hierarchy with respect to their deductive strength

\[ [\dot{u} : P \rightarrow P] < [\Box u : P \rightarrow P] < [\Box\Box u : P \rightarrow P] < \ldots < [\Box^k P \rightarrow P]. \]

Fedor Pakhomov
*Complexity of Fragments of the Logic GLP*

I will discuss the computational complexity of some fragments of the logic GLP. The considered fragments are the variable-free fragment and its subfragments given by the restriction on the number of different modality symbols in a formula. The whole variable-free fragment is PSPACE complete. All of the considered subfragments are polynomial-time decidable.
Wolfram Pohlers

*Ordinal analysis and Hilbert’s programme*

There are two main aspects of Hilbert's programme. Establishing the consistency of Analysis (i.e., second order arithmetic) by finitistic means and the elimination of ideal elements. Gentzen's contributions to Hilbert's programme—in which he launched the branch of proof theory which we today call ordinal analysis—originally aimed at the first aspect of Hilbert's programme. However, by Gödel's incompleteness theorems we learned that this aspect is unattainable in full rigidity. In this talk we want to argue that Gentzen's ordinal analysis and its later enhancements contribute essentially to the aspect of "elimination of ideal elements" in Hilbert's programme.

Michael Rathjen

*Power Kripke-Platek set theory, ordinal analysis and global choice*

KP and extensions via axioms asserting the existence of many admissible sets have been studied intensively in admissible proof theory. Augmenting KP by the powerset operation either by adding the axiom or treating the operation as a primitive (Power KP) leads to theories that are to some extent amenable to proof-theoretic techniques, thereby providing some form of ordinal analysis. The proof-theoretic techniques enable one to considerably strengthen classical results. Moreover they entail conservativity results with respect to the axiom of global choice and the calculus of constructions with one universe.

Stephen G. Simpson

*Reverse mathematics, ordinal numbers, and the ACC*

In abstract algebra, a ring is said to satisfy the ACC (ascending chain condition) if it has no infinite ascending sequence of ideals. According to a famous and controversial theorem of Hilbert, 1890, polynomial rings with finitely many indeterminates satisfy the ACC. There is also a similar theorem for noncommuting indeterminates, due to J. C. Robson, 1978. In 1988 I performed a reverse-mathematical analysis of the theorems of Hilbert and Robson, proving that they are equivalent over RCA₀ to the well-orderedness of (the standard notation systems for) the ordinal numbers ωω and ωωω respectively. Now I perform a similar analysis of a theorem of Formanek and Lawrence, 1976. Let $S$ be the group of finitely supported permutations of the natural numbers. Let $K[S]$ be the group ring of $S$ over a countable field $K$ of characteristic 0. Formanek and Lawrence proved that $K[S]$ satisfies the ACC. All of these results concerning the ACC involve well partial ordering theory. I now prove that the Formanek/Lawrence theorem is equivalent over RCA₀ to the well-orderedness of ωωω. The proof involves an apparently new, combinatorial lemma concerning Young diagrams. I also show that, in all of these reverse-mathematical results, RCA₀ can be weakened to RCA₀*.

This recent work was done jointly with Kostas Hatzikiriakou.

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Paul Shafer
Exploring randomness, diagonally non-recursiveness, and Ramsey-type combinatorial principles in reverse mathematics

This is joint work with Laurent Bienvenu (LIAFA, Paris 7) and Ludovic Patey (PPS, Paris 7).

Stephen Flood introduced Ramsey-type weak König’s lemma (RWKL), a simultaneous weakening of weak König’s lemma (WKL) and Ramsey’s theorem for pairs and two colors. An instance of RWKL is an infinite binary branching tree T, and a solution to this instance is not an infinite path through T as with WKL, but an infinite set consistent with being a path through T. Among several results, Flood proved that RWKL implies DNR, but he left open whether or not the reverse implication holds.

Motivated by Flood’s question of whether or not DNR implies RWKL, we introduced Ramsey-type variations of other consequences of WKL and studied their relationships with DNR. Specifically, we studied RSAT, a Ramsey-type variant of the Boolean satisfiability problem (i.e., compactness in propositional logic); RWWKL, a Ramsey-type variant of weak weak König’s lemma (WWKL); and RCOLOR(k), a Ramsey-type variant of the fact that every locally k-colorable graph is k-colorable. We found that RSAT is equivalent to RWKL and that RWWKL is equivalent to DNR (the latter equivalence reflecting results of Kjos-Hanssen and of Greenberg and Miller). We also proved that there is a recursive instance of RCOLOR(2) such that the measure of oracles that compute solutions to the instance is zero. It follows that WWKL does not imply RCOLOR(2). Combining this result with the well-known fact that WWKL implies DNR and the fact that RWKL implies RCOLOR(2) answers Flood’s question: DNR does not imply RWKL.

Daniyar Shamkanov
Nested Sequents for Provability Logic GLP

We present a proof system for the provability logic GLP in the formalism of nested sequents and prove the cut elimination theorem for it. As an application, we obtain the reduction of GLP to its important fragment called J syntactically.