Mathematical Methods for Physics II (2006/07)

Course:	MMFII: Mathematical Methods for Physics II
Class:	No. 9 at Physics Building
Group:	III
Exams:	P1 23-01-08; P2 03-06-08; F1 24-06-08; F2 12-09-08
	All exams will be at the Main Hall of Physics Building starting at 9:30

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	Theory and Problems
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Methodology:

This is a two semester course which content is to be given in a set of 12 docent credits, 7,5 for theoretical contents plus 4,5 for problems. Therefore this is a four hour per week course. The student is supposed to be familiar with basic knowledge from infinitesimal calculus and linear algebra. Theoretical lessons will be devoted to present the fundamental methods and results that will help us to solve different problems. Practical lessons will be devoted to solve some of these problems, students will be welcome to solve on the blackboard some of these problems.

Course Objectives:

The main objective of this course is to familiarize our students with some of the most common mathematical techniques in the Physical Sciences. More particularly, our objectives are:

- To learn the basic concepts and techniques relative to the ordinary differential equations. We will learn how to solve particular cases of first and second order equations at the time that we will study more advanced techniques as the resolution of differential equations by power series or the use of the Laplace transform for solving equations.
- To learn the multiple and surface integration techniques with application to conservative fields and potential theory.
- To learn and understand the importance and beauty of complex functions and their interplay with the power series. We will learn how to evaluate definite integrals by use of the residue theorem.
- To learn and appreciate the importance and beauty of Fourier series, Fourier transforms, and their relatives, and how these tools help solving certain types of PDEs. We

will give some examples of applications of these techniques such as data compression, where the boundary conditions and geometry are also very important.

- To introduce the student to some of the most fundamental and basic PDEs as the heat equation, the wave equation or the Laplace equation.

Topics:

Ordinary Differential Equation, Vectorial Analysis, Functions of Complex Variable, Fourier Series and Fourier Transforms, some basic PDEs, basic notions on Numerical Analysis

Contents:

The course contents are divided into the 6 following blocks:

Block I: Ordinary differential equations.

1. **Introduction to differential equations.** What is a differential equation? Notion of solution. Examples. Orthogonal trajectories. Picard's Theorem. Applications.
2. **First order differential equations.** Separation of variables. Linear equations. Exact and homogeneous equations. Integrating factors. Change of variables and Bernuilli equations. Reduction of order.
3. **Second-order linear differential equations.** Linear equations. Structure of the solutions. Homogeneous equation with constant coefficients. Methods of undetermined coefficients and variation of parameters.
4. **Series solutions of differential equations and Special functions.** Series solutions of first-order differential equations. Regular and singular points of an equation of second order. Equations of Bessel and Legendre.
5. **The Laplace transform.** Introduction to integral transforms. Definition and properties of the Laplace transform. Application to differential equations. Convolution. Step and impulse function.

Block II: Vector analysis.

1. **Multiple integrals.** Riemann integral. The Cavalieri principle. Double integral on rectangles, Fubini Theorem. Double integral on general regions. Triple integral. Applications to areas, volumes, mass, potentials, moment problems,...
2. **Change of variables.** Polar coordinates. Change of variables in the plane. Spherical and cylindrical coordinates. Change of variable theorem in $\mathbb{R}^3$.
3. **Line integrals.** Curves. Tangent vectors. Length. Orientation. Line integral of a scalar field. Line integral of a vector field. Green's theorem in the plane.
4. **Surface integrals.** Surfaces. Normal vector. Element of area. Orientation. Surface integrals of scalar and vector fields. Flow. Stokes' Theorem. Gauss' Divergence Theorem.

Block III: Functions of a complex variable.

1. **Complex numbers.** Complex numbers. Complex functions. Continuity.
2. **Complex derivative.** Complex derivative. Cauchy-Riemann's Theorem. Harmonic functions. Elemental complex functions and derivatives.
3. **Complex integral.** Definition and primitive theorem. Cauchy Formula. Cauchy integral formula. Applications.
4. **Representation in series.** Power series. Taylor series. Laurent series.
5. **Residues.** Singularities and residues. Poles. Residue at simple and multiple poles. Residue at infinity. Applications to integrals and series.

Block IV: Fourier series and transform.

1. **Fourier series.** Introduction. Periodic functions. Trigonometric series. Fourier coefficients and Fourier series. Pointwise convergence of a Fourier series. Fourier series of odd and even functions. Other intervals.
2. **Cuadratic convergence.** Orthogonality. Euclidean norm. Convergence in cuadratic mean. Orthogonal systems. Parseval's Theorem.
3. **Fourier transform.** Complex Fourier series. Definition of Fourier transform. Calculus of transforms. Properties. Convolution. Parseval's Theorem. Dirac's Function. Applications.

Block V: Introduction to partial differential equations.

1. The wave equation. The heat equation. Laplace's equation. Separation of variable. Unicity of solutions.

Block VI: Numerical calculus.

1. Errors. Newton's method. Trapezoid rule. Simpson's rule. Euler's method. Runge-Kutta's method.

Textbook:

- Mary L. Boas: Mathematical Methods in the Physical Sciences, 2nd Edition, John Wiley & Sons, 1983, ISBN: 0-471-004409-1. This text will be followed as textbook although lessons will be self-contained. We will cover different parts of Chap. 2, 5, 6, 7, 8, 12, 13, 14 and 15 of this book.
- Roel Snieder: A Guided Tour of Mathematical Methods for the Physical Sciences, Cambridge University Press 2001. This text is of great beauty and highly recommended for extra readings, some motivating examples could be taken from this book.

Other references:

- Arfken-Weber: Essential Mathematics for Physicists, Elsevier Science, San Diego 2004.
- J. E. Marsden, A. J. Tromba: Cálculo vectorial, Cuarta Edición, Editorial Addison Wesley, 1998.
- K. F. Riley, M. P. Hobson and S. J. Bence: Mathematical Methods for Physics and engineering, Segunda Edición , Cambridge University Press 2002.
- J. San Martín Moreno, V. Tomeo Perucha e I. Uña Juárez: Métodos Matemáticos, Editorial Thomson 2004.
- G. F. Simmons: Ecuaciones Diferenciales, Segunda Edición, Editorial McGraw Hill 1993.

Attendance:

Formal attendance will not be taken. However, whether you are able to attend class or not, you are responsible for all material presented in class as well as any work that may be due. It is your responsibility to find out what happened if you miss class.

Class Web Page:

We will maintain the Web pages for this course (http://www.personal.us.es/espinola/mmfII/mmfI
All homework assignments and important announcements will be posted on this page. Please check this page regularly.

Grading Scheme:

- 15 % Homework (each two Fridays beginning October 5)
- 85 % Partial Exam (1P 9:30 January 23, 2008; 2P 9:30 June 3, 2008)

Homework:

We will assign homework problems each two Fridays from October 6, which can be seen at Homework Page (http://www.personal.us.es/espinola/mmfII/homeworks.htm). The due date is the Friday of two weeks later at the beginning of class. Homeworks written in Spanish are acceptable. LATE HOMEWORK WILL NOT BE ACCEPTED. All homework must be NEAT, ACCURATE, and LEGIBLE. You are encouraged to write in complete sentences. You can be penalized if your work cannot be followed. Getting the correct answer is only part of the problem. A subset of these problems will be graded, and returned on the following Friday at the end of class. We will not include the score of the two worst performed homeworks when computing your grade.

Note: Criteria from EEES (Espacio Europeo de Educación Superior, http://eees.universia.es/) establishes that for each hour of theory you need an hour and a half of study, this means that for our subject you are supposed to work in between four and five hours per week to mature the theoretical contents. Also EEES establishes that for each hour of problem class you are supposed to study another hour. Homeworks will not make you exceed these estimates.

Exams:

There will be two partial exams each one at the end of the corresponding semester. Each student will obtain a mark after the second partial exam for the whole course.

If you do not pass then you have a second option at June Final Exam. If you pass but you would like to improve your mark then you can choose to take the Final Exam. (In this case, your final mark the highest between your mark in the final and your mark from the course.)

If you pass one semester then you can choose to take only the part you failed in the Final Exam, in this case your final mark will be the average between the mark of the passed partial and the mark from the final. This average will ONLY apply if you obtain a mark of at least 4 (out of 10) points in the final.

Those who did not pass the course by July will have another chance at September Final Exam. Cancelation of passed partials DO NOT apply for September Final Exam.

September 20, 2007

Rafael Espínola García