Quantum entanglement in elliptical quantum corrals

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1. Introduction

Modern laboratory techniques have allowed an important progress in the comprehension of quantum phenomena in solid state structures at the nanoscopic scale. This has been possible thanks to the design, modelling and fabrication of devices for quantum confinement and the control of atoms, photons and electrons. One of its consequences has been to motivate the development of novel applications in nanotechnology, particularly in quantum electronics.

At the same time, an innovative theory such as quantum information has evolved into a new paradigm in computation and communication that has overcome the limits of classical information [1]. While classical bits in binary logic take two possible values, zero and one, quantum computation is based on new quantum units of information or qubits, which represent a superposition of ones and zeros. Its multiple applications (computation, communication, cryptography or quantum teleportation, among others) require a controlled manipulation of light and matter at the quantum level.

In this sense nanostructured materials play an important role [2,3]. Among these we can mention the so-called quantum corrals, where the electrons remain quantically confined by means of a geometrical array of adatoms deposited on the surface of noble metals, forming a nearly closed structure. These systems have allowed the observation of surprising effects such as quantum mirages [4,5], where the introduction of an impurity on one of the foci of an elliptical structure formed a ghost image in the empty one. These systems are excellent candidates for technological applications in quantum information and spintronics [6–8]. For example, by adding a magnetic atom on one focus the quantum mirage could be used as a geometric protection of magnetic qubits against decoherence. In addition, quantum entanglement can be feasible if two magnetic impurities are added at the foci. Also feasible is the implementation of logical gates of one and two qubits. In this case, these corrals could serve as elementary prototypes of quantum computers.

In a recent work we have shown that, when a magnetic impurity is added to a focus, an image is formed at the other focus, which can be observed in the local density of states (DOS) when the Kondo effect is operative [9,10]. This implies that in any system with focalizing properties, two impurities which are located at relatively large distances could strongly interact. We have studied the problem with two impurities [11–13] where we have calculated the time response to localized stimulations and the degree of quantum entanglement for different partitions of the system.

In this work we present results for the quantum entanglement between two localized magnetic impurities which interact antiferromagnetically with the itinerant surface electrons of the...
ellipse. We find that, when this interaction is weak, the spins are strongly entangled in a singlet state while they are disentangled from the elliptical degrees of freedom. For large interactions, instead, the spins decorrelate while increasing their entanglement with the electrons. This state can be characterized by measuring the local density of states at the foci.

2. The model

In Fig. 1 we sketch the model used, which consists of a hard-wall ellipse with non-interacting electrons, which interact via an antiferromagnetic superexchange interaction with two localized spins, which are disentangled from the Fermi levels to resemble the experiments. However, other Fermi levels lead to similar results provided the weight of the eccentricity to coincide with the experiments. In the basis of eigenstates of the ellipse, these local densities of states at the foci are appreciable. For large interactions, instead, the spins decorrelate while increasing their entanglement from the elliptical degrees of freedom. For large interactions, the localized spins are no longer in a singlet and the entropy or entanglement increases. In the largest entangled case we have in focus [11], then $S_{\text{max}}$ can be thought of as due to an effective RKKY state and the ground state consists of a direct product of the non-interacting state and the ground state of the non-interacting ellipse (Fig. 2). For the even case and small interaction, the entropy has a simpler form

$$ S = -\sum_i \omega_i^h \log_2 \omega_i^h = -\sum_i \omega_i^o \log_2 \omega_i^o. $$

The quantum entropy is positive and lower than the smallest of the dimensions of A and B. If $|\psi\rangle$ is formed as a direct product of states in A and B ($\omega_A^h = \omega_B^h = 1$) then $S = 0$ and there is no entanglement. On the contrary, when all states in A combine with all states in B (completely mixed density operator, $\omega_{AB} = 1/d$, where $d$ is the smallest dimension of the Hilbert spaces of A and B), then $S_{\text{max}} = \log_2(d)$.

We will study a particular partition which consists of considering the localized spins separated from the ellipse (Fig. 1b). This helps us understand the entanglement of the spins and the rest of the ellipse while varying the magnetic interaction.

For this partition we find that the entropy increases with $J$ and has a slightly different behaviour for even and odd particles in the ellipse (Fig. 2). For the even case and small interaction, the entropy is negligible because the spins are locked into a singlet state [11] and the ground state consists of a direct product between this singlet and the ground state of the non-interacting ellipse. This state can be thought of as due to an effective RKKY interaction via the itinerant electrons, which is enhanced due to confinement. When $J$ increases, again more states get involved, the localized spins are no longer in a singlet and the entropy or entanglement increases. In the largest entangled case we have in A a product of all four spin states ($\uparrow \uparrow \uparrow \uparrow, \downarrow \downarrow \downarrow \downarrow$), which
appear with equal probability. The spins are completely decoupled and form local Kondo singlets with the electrons.

For the odd case and small $J$, the spins are in a mixture of mainly two triplet states because we are considering the $S_z = 1/2$ subspace. Again, for large $J$, the results match the even case and have maximum entropy ($S = \log_2 4 = 2$).

4. Local density of states

As a means to detect whether the system is in the local Kondo regime we can resort to the observation of the local densities of states defined as

$$\rho(\omega) = \frac{-1}{\pi} \lim_{\eta \to 0} \text{Im} \ G(\omega + i\eta + E_0),$$

$$\rho^*(\omega) = \frac{-1}{\pi} \lim_{\eta \to 0} \text{Im} \ G^*(\omega + i\eta + E_0),$$

where

$$G(z) = \langle \psi_0 | c_i^\dagger (z - H)^{-1} c_i | \psi_0 \rangle,$$

$$G^*(z) = \langle \psi_0 | c_i^\dagger (z - H)^{-1} c_i^\dagger | \psi_0 \rangle,$$

$\omega$ is the energy, $i = 1, 2$ are the foci of the ellipse, $E_0$ is the ground state energy. The quantities defined above correspond to the photoemission and inverse photoemission spectra, respectively.

We present our results in Fig. 3. Here we can distinguish two different regimes. For small $J$, the spectra consist mainly of the discrete levels of the isolated ellipse. However, when the antiferromagnetic superexchange interaction $J$ increases ($J/2t^* > 1$), a pseudogap or a reduction of the local DOS starts developing, showing the transition to a different state. This reduction is a consequence of the Fano interference between the states of the ellipse and the localized spin when the many-body Kondo singlet is formed [14], thus signalling the mutual decoupling of the two impurities and the onset of the local Kondo state.

5. Conclusions

We have studied an elliptical confined system with focalizing properties in the presence of two localized impurities which interact antiferromagnetically (via $J$) with the itinerant electrons of the corral. We found a different behaviour for even and odd fillings in the ellipse which shows up in the character of the ground state and the excitations. For the even-particle case and small interaction $J$, both localized spins are entangled in a (quasi)-singlet state. When increasing the antiferromagnetic interaction $J$, an interesting feature arises which resembles the RKKY–Kondo-like transition occurring in the two-impurity system: while the localized spins form a singlet state between them for small interactions, this coupling decreases for larger $J$ giving rise to an on-site singlet correlation between the spin and the itinerant electron. For odd number of electrons the effective interaction is ferromagnetic in the ground state. The character of this interaction can be controlled by changing the chemical potential of the system.

In this work we have also analysed the entanglement and von Neumann entropy for a particular partition of the system in the ground state, which offers us an alternative perspective on the problem. When the impurities are locked in a singlet state their entanglement with the ellipse is negligible, while it grows to its maximum value when they decouple from each other and form a local Kondo state with the electrons.

For small to moderate values of $J$ ($J/2t^* \leq 1$) we expect the main results synthesized above to hold in the case of more realistic models of quantum ellipses which include tunnelling of the electrons in open corrals and inelastic processes with bulk electrons. In this parameter range the broadening of the relevant energy levels is smaller than their separation [10,5]. When larger interactions are included, higher levels which are more hybridized take part, and models including these processes should be considered.

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References


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