PhD thesis defense:

Modelling and Simulation of Laser Dynamics with Cellular Automata on Parallel and Distributed Computers

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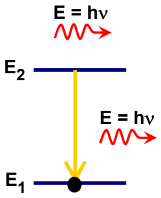
14 December 2009

QUESTION 1

LASER

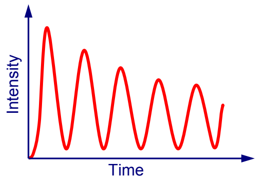
Mechanism

Stimulated emission



Phenomenology

- Laser threshold
- Relaxation oscillations



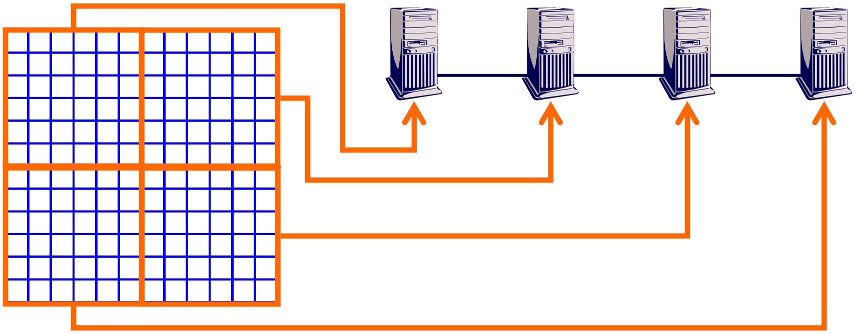
- Other

Is it possible to model a laser using a cellular automaton (CA) that reproduces the phenomenology?

Interest of a CA laser model

- **New methodological approach**
- Reinforces the vision of **laser** as a **complex system**
- **Applications**:
  - Lasers ruled by equations with convergence problems
  - Difficult boundary conditions
  - Very small optoelectronic devices
  - Study interesting problems: chaos or emergent phenomena in lasers
  - Parallel model

QUESTION 2



CA laser model

Heterogeneous parallel and distributed computers

Is it possible to take advantage of the intrinsic parallel nature of CA to develop efficient parallel implementations?

Outline

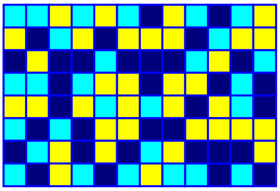
- Modelling laser dynamics with a cellular automaton
    1. Introduction: Cellular Automata (CA) and laser dynamics
    2. CA model for laser dynamics
    3. Simulation results

- Efficient implementation on parallel and distributed computers
    1. Parallelization
    2. Execution on dedicated clusters
    3. Execution on non-dedicated clusters with load balancing

- Conclusions

Cellular automata (CA)

- A class of mathematical systems:
    - **Space** and **time**: **discrete**
    - Each cell: **discrete states**
    - **Local interactions**: each cell interacts only with its **neighbours**
    - Discrete dynamics: **evolution rules**
    - **Parallel** nature

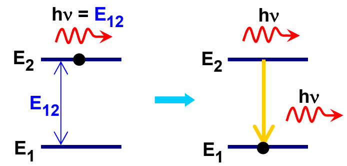


- Models for **complex systems** → **emergent behaviours**

- **Applications**:
    - **Natural sciences**: models in physics, chemistry, biology, geology...
    - Mathematics
    - Theoretical computer science
    - Engineering

Laser: physical processes

- **Laser**: Device that generates electromagnetic radiation based on the **stimulated emission process**:



- This process competes with **absorption**
- Normally: lower level more populated ⇒ absorption has greater probability than emission
- Laser mechanism: **energy pumping process** ⇒ **population inversion**

⇓

- An incoming photon with $h\nu=E_{12}$ can give rise to a cascade of **stimulated coherent photons**

Standard description of laser dynamics: laser rate equations

$$
\begin{cases}
\dfrac{dn(t)}{dt} = KN(t)n(t) - \dfrac{n(t)}{\tau_c} \\[2ex]
\dfrac{dN(t)}{dt} = R - \dfrac{N(t)}{\tau_a} - KN(t)n(t)
\end{cases}
$$

n(t) → number of laser photons

N(t) → population inversion

τ_c → decay time of photons in the cavity

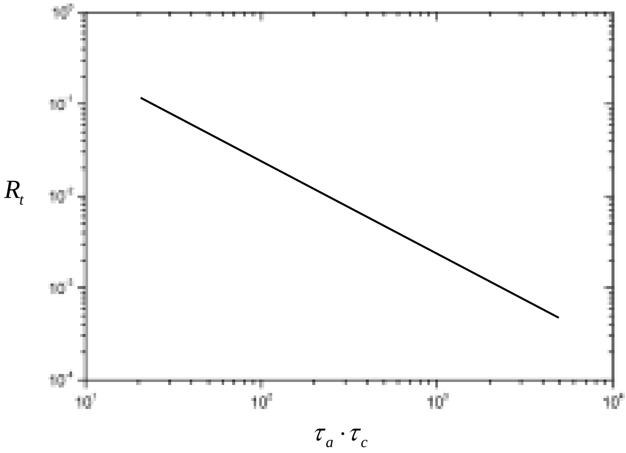
τ_a → decay time of the upper laser level

R → Pumping rate

K → Coupling constant

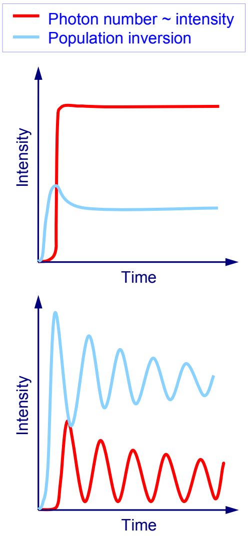
Laser dynamics:
Dependence of threshold pumping rate on laser parameters

- From the **laser rate equations**: $R_t = \dfrac{1}{K\,\tau_a \tau_c}$



Laser dynamics: behaviours

- **Laser operation** → different characteristic **dynamic behaviours** are shown depending on the type of laser (or on the laser parameters):
  - **Constant regime:**
    - After an initial transient, the system reaches an stationary constant behaviour
  - **Relaxation oscillations (or laser spiking):**
    - Correlated large amplitude damped oscillations in the photon number and population inversion

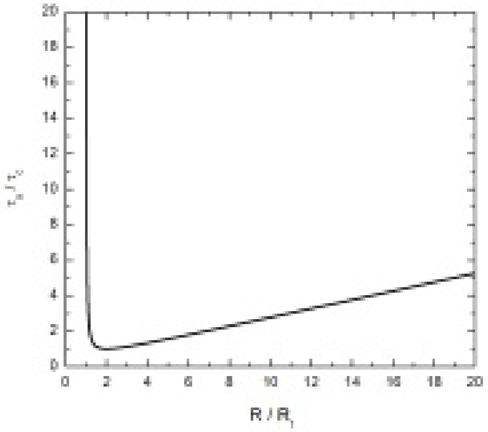


Laser dynamics:
Dependence of behaviour on laser parameters

- **Laser rate equations** → depending on parameters values, **2 main behaviours**:
  - Oscillatory
  - Constant regime

} **Theoretical stability curve** ⟶

$$\frac{\tau_a}{\tau_c} = \frac{\left(\frac{R}{R_t}\right)^2}{4\left(\frac{R}{R_t} - 1\right)}$$



τ_a → Life time of excited electrons

τ_c → Life time of laser photons

R → Pumping rate

CA model for laser dynamics (1)

2D, multivariable and partially probabilistic CA:

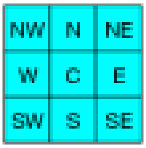
- **Cellular space**: 2-dims. square lattice with periodic boundary conditions
- **States of the cells**: each cell has four variables associated:

$$\left\{ \begin{array}{ll} a_{\vec{r}}(t) \in \{0,1\} & \rightarrow \textbf{State of the electron} \\ c_{\vec{r}}(t) \in \{0,1,2,\ldots,M\} & \rightarrow \textbf{Number of photons} \\ \tilde{a}_{\vec{r}}(t) \in \{0,1,2,\ldots,\tau_a\} & \rightarrow \text{Time since electron in upper laser state} \\ \tilde{c}^{k}_{\vec{r}}(t) \in \{0,1,2,\ldots,\tau_c\} & \rightarrow \text{Time since photon k was created} \end{array} \right.$$

(in cell $\vec{r} = (i, j)$ at time t)

- **Neighbourhood:**

"Moore neighbourhood":
Each cell has
nine neighbours:



NW	N	NE
W	C	E
SW	S	SE

$$\Gamma_{\vec{r}}(t) = \sum_{\vec{r}' \equiv neighb.(\vec{r})} c_{\vec{r}'}(t)$$

CA model for laser dynamics (2)

- **Transition function**:
  - **R1- Pumping**: If $\{\, a_{\vec{r}}(t)=0 \,\} \;\rightarrow\; a_{\vec{r}}(t+1)=1$ with a probability λ
  - **R2- Stimulated emission**: If $\{\, a_{\vec{r}}(t)=1,\;\; \Gamma_{\vec{r}}>\delta \,\} \;\rightarrow\; \begin{cases} c_{\vec{r}}(t+1)=c_{\vec{r}}(t)+1 \\ a_{\vec{r}}(t+1)=0 \end{cases}$
  - **R3- Photon decay**: Photon is destroyed τ_c time steps after it was created
  - **R4- Electron decay**: Electron decays τ_a time steps after it was promoted
  - **R5-** Evolution of temporal variable $\tilde{a}_{\vec{r}}(t)$: counts number of time steps since an electron is promoted to upper state.
  - **R6-** Evolution of temporal variable $\tilde{c}_{\vec{r}}^{\,k}(t)$: counts number of time steps since a photon is created.
  - **R7- Random noise photons**: $c_{\vec{r}}(t+1)=c_{\vec{r}}(t)+1$ for ~ 0.01% of total cells

Simulations

- **Initial state**: $a_{\vec{r}}(0)=0,\quad c_{\vec{r}}(0)=0,\quad \forall\vec{r}$ except small fraction of noise photons
- The **system evolves** by the application of the transition rules
- In each time step, we measure:
  - **n(t)**: Total number of laser photons
  - **N(t)**: Total number of electrons in upper laser state ≡ population inversion
- System → **3 parameters: { λ, τ_c , τ_a }**:
  - λ → Pumping probability
  - τ_c → Life time of laser photons
  - τ_a → Life time of excited electrons
- System size used: normally 400×400 cells

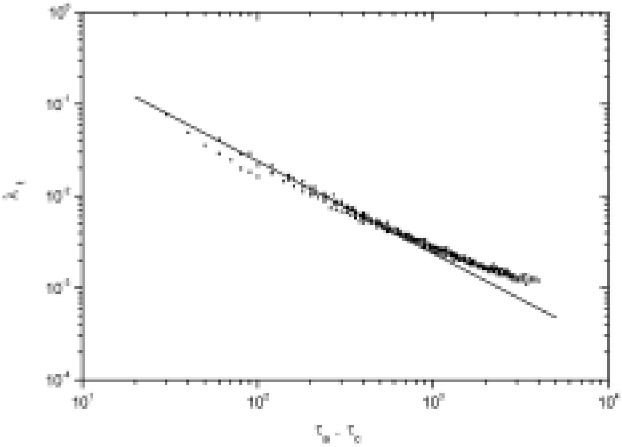
Simulation results:
Dependence of threshold pumping probability on laser parameters

- From the **laser rate equations**:

$$\lambda_t = \frac{1}{C\,\tau_a \tau_c}$$

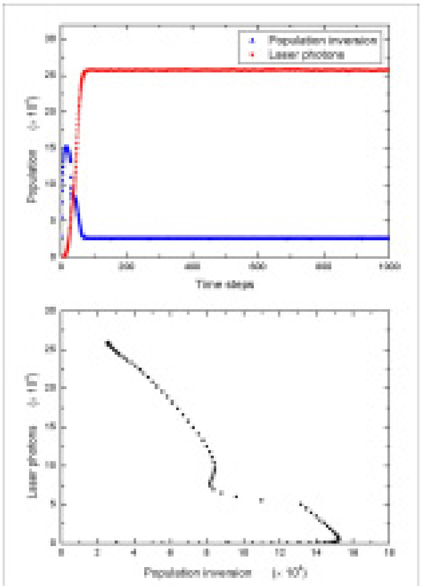
$$(\lambda \propto R)$$

- From the **simulation**:

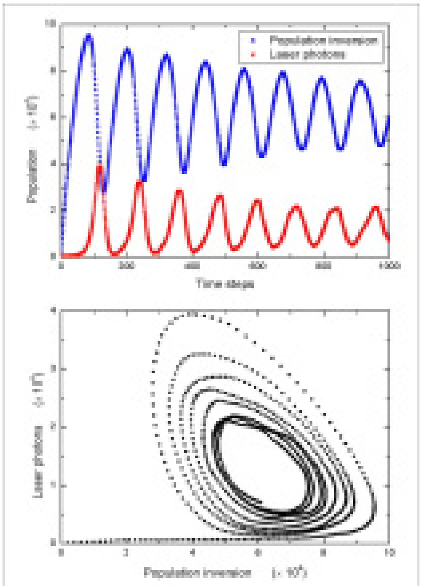


Simulation results:
Lasers behaviours

(a): Constant regime



(b): Relaxation oscillations (laser spiking)

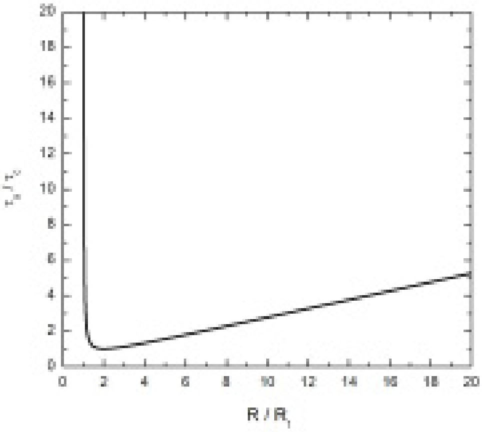


Simulation results:
Dependence of behaviour on laser parameters

- **Laser rate equations** → depending on parameters values, **2 main behaviours**:
  - Oscillatory
  - Constant regime

  } **Theoretical stability curve** →

$$\frac{\tau_a}{\tau_c} = \frac{\left(\frac{R}{R_t}\right)^2}{4\left(\frac{R}{R_t} - 1\right)}$$



τ_a → Life time of excited electrons

τ_c → Life time of laser photons

R → Pumping rate

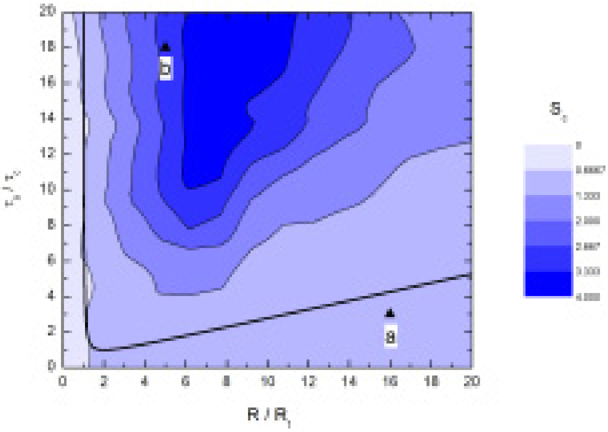
Simulation results:
Dependence of behaviour on laser parameters

- **Laser rate equations** → depending on parameters values, **2 main behaviours**:
  - Oscillatory
  - Constant regime

  } **Theoretical stability curve** →

$$\frac{\tau_a}{\tau_c} = \frac{\left(\frac{R}{R_t}\right)^2}{4\left(\frac{R}{R_t} - 1\right)}$$

- **Simulations** → **Shannon's entropy** of temporal distribution of n(t) and N(t): **fingerprint of oscillations**



$$S = -\sum_i f_i \log_2 f_i$$

τ_a → Life time of excited electrons

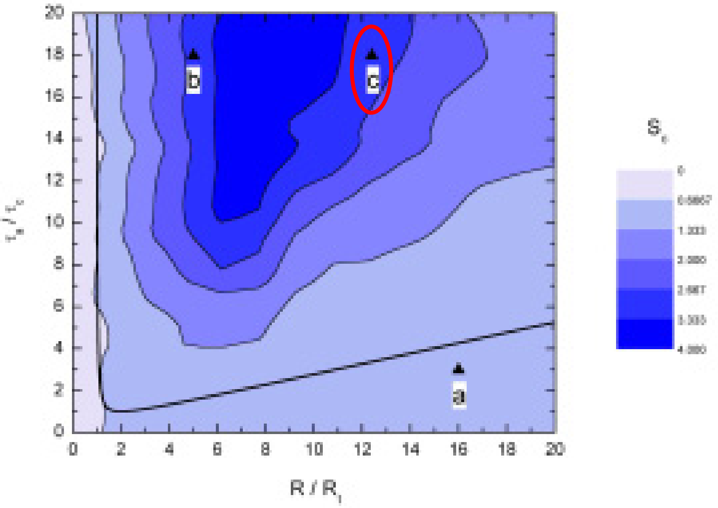
τ_c → Life time of laser photons

R → Pumping rate

λ → Pumping probability

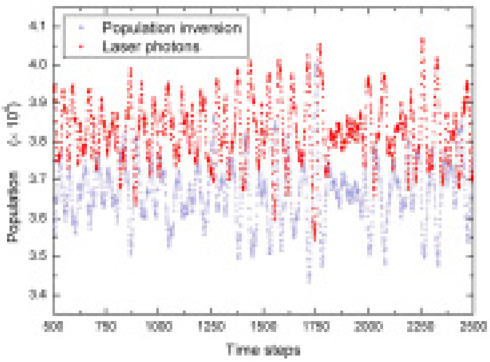
(with $\frac{R}{R_t} = \frac{\lambda}{\lambda_t}$)

Simulation results:
Irregular oscillations



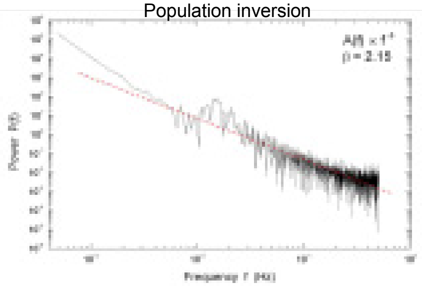
Simulation results:
Irregular oscillations

(c): Complex behaviour showing irregular oscillations:

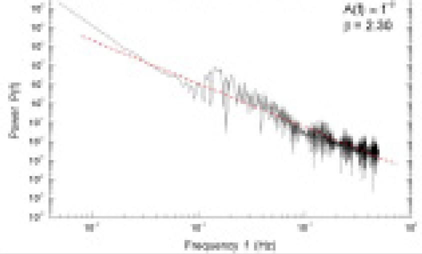


Power spectrum: $1/f^{-2}$ noise

Population inversion



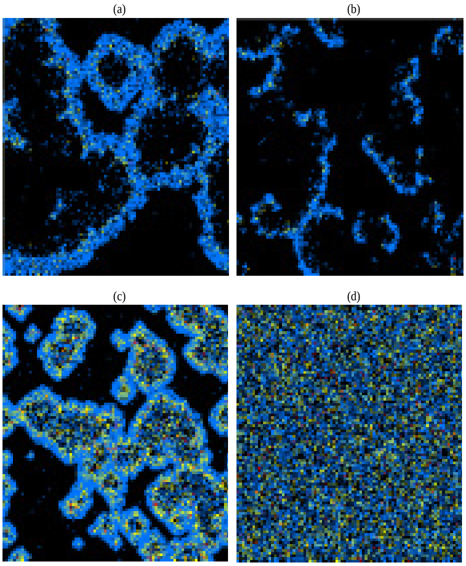
Laser photons



Simulations results:
Spatio-temporal patterns

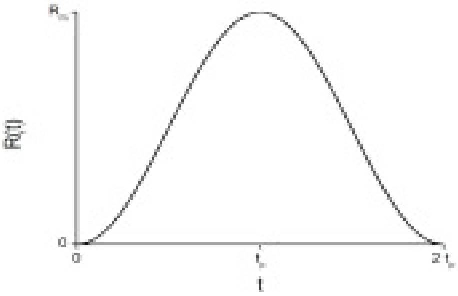
Oscillatory behaviour →

Constant regime →



Simulation of pulsed pumped lasers

- **Pumping rate R(t) → time dependent pulsed form**:



$R(t) = R_m \, \Phi(t)$

(Width: 2 t_p)

- **Laser rate equations:**

$$\begin{cases} \dfrac{dn(t)}{dt} = KN(t)n(t) - \dfrac{n(t)}{\tau_c} \boxed{+ \varepsilon \dfrac{N(t)}{\tau_a}} \\ \dfrac{dN(t)}{dt} = \boxed{R(t)} - \dfrac{N(t)}{\tau_a} - KN(t)n(t) \end{cases}$$

→ spontaneous emission process

time dependent pumping rate

(ε → fraction of radiative decay processes that create a photon in the laser mode)

CA model for pulsed pumped lasers

- **Modified transition function**:
  - **R1- Pumping**: If $\{ a_{\vec r}(t) = 0 \} \rightarrow a_{\vec r}(t+1) = 1$ **with a probability: $\lambda(t) = \lambda_{max}\,\Phi(t)$**
  - **R2- Stimulated emission**: If $\{ a_{\vec r}(t) = 1, \ \Gamma_{\vec r} > \delta \} \rightarrow \begin{cases} c_{\vec r}(t+1) = c_{\vec r}(t) + 1 \\ a_{\vec r}(t+1) = 0 \end{cases}$
  - **R3- Photon decay**: Photon is destroyed τ_c time steps after it was created
  - **R4- Electron decay**: Electron decays τ_a time steps after it was promoted **and a new photon will be created in that cell with a probability θ**
  - **R5-** Evolution of temporal variable $\tilde{a}_{\vec r}(t)$: counts number of time steps since an electron is promoted to upper state.
  - **R6-** Evolution of temporal variable $\tilde{c}^{k}_{\vec r}(t)$: counts number of time steps since a photon is created.
  - **No random noise photons are created**

⇓

Spontaneous emission process is associated with the decaying of the population inversion

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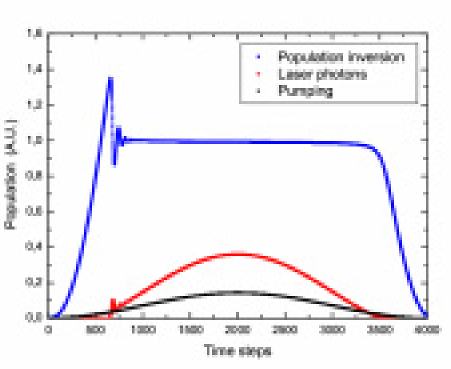
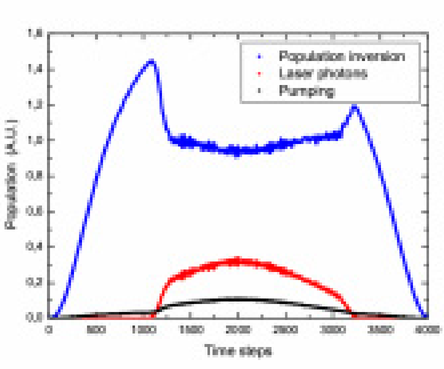
Pulsed pumped lasers results: differential equations versus CA

Numerical integration of differential equations:



(Pumping: Value of R(t))

CA model simulation:



(Pumping: Number of electron cells pumped by rule R1 in each time step)

Outline

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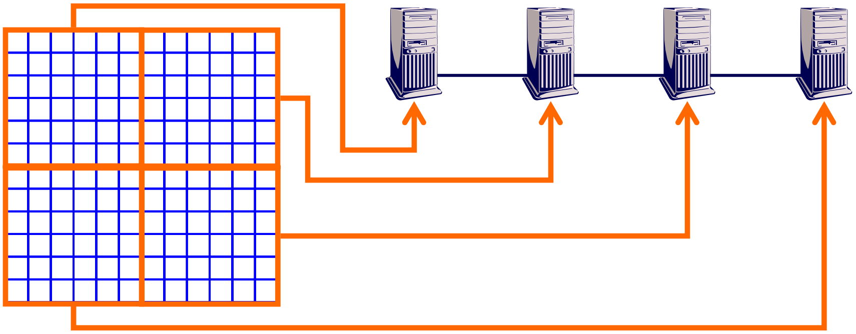
- Efficient implementation on parallel and distributed computers
    1. Parallelization
    2. Execution on dedicated clusters
    3. Execution on non-dedicated clusters with load balancing
- Conclusions

QUESTION 2



CA laser model

Heterogeneous parallel and distributed computers

Is it possible to take advantage of the intrinsic parallel nature of CA to develop efficient parallel implementations?

CA models on parallel computers

CA models:

- Very **suitable** to be implemented efficiently on **parallel computers**:
  - **Intrinsic parallel nature**: Evolution rules → in parallel to all the cells
  - **Local nature**: Evolution rules → local

⇒ The system can be **split into partitions**:

- Run on different processors
- Communication flow between processors can be kept low

Parallel version of the CA laser model

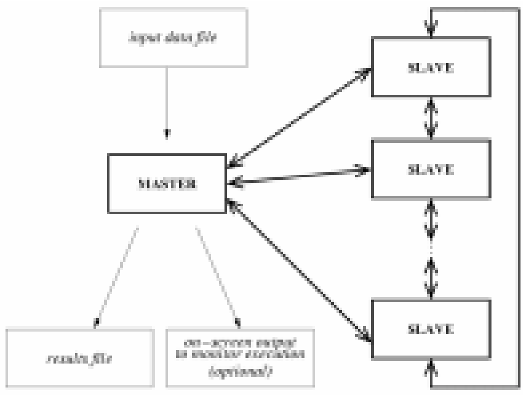
- For **detailed** laser dynamics **simulations** (fine grained 2D or 3D models):
  Large execution time and memory required ⇒ **parallel implementation is necessary**
- **Difficulties** for good performance: CA model **only partialy uncoupled**:
  - Synchronous CA ⇒ Nodes must exchange information after each time step
  - All the nodes must have finished an iteration before the next one can be started
  - Parallel application performance: limited by the slowest task
- Such a model → good performance on shared memory parallel computers
- **Our contribution:**

... Is it feasible to run a parallel version of the model on parallel and distributed computers ?

(On dedicated or heterogeneous non-dedicated clusters)

Parallel implementation

- For **distributed-memory parallel computers**, using **message passing** (PVM)
- Following:
  - **Master-slave programming model**
  - **Data decomposition** methodology for workload allocation

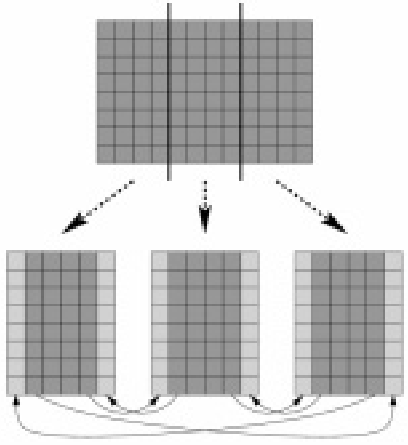


Parallel implementation

- **1D Domain decomposition** (stripes):

  ↓
  - minimize number of send/receive calls
- Ghost cells.
- Laser CA model: **only neighbours' state needed :** $c_{\vec{r}}(t)$ (# of photons)

  ↓
  - minimize communications



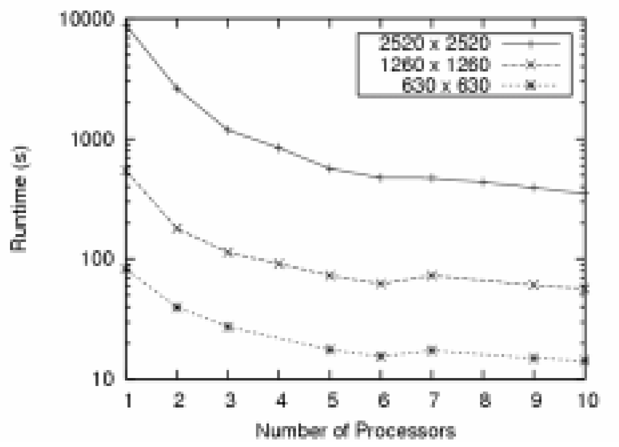
Execution of parallel model

- **Heterogeneous PC cluster** - 10 nodes Intel Pentium-4:
  - **6 "fast" nodes** (2.7 GHz): simulations with 1 to 6 nodes
  - **4 "slow" nodes** (1.8 GHz): simulations with more than 6 nodes
  - O.S: Linux (Rocks distribution, based on Red Hat)
  - RAM Memory: 512 MB each node
  - Communications: Fast-ethernet 100 Mbps switch

- To **evaluate performance**:
  - Running the same experiment for **different**:
    - **Number of nodes**
    - **System sizes**

Performance analysis: runtime

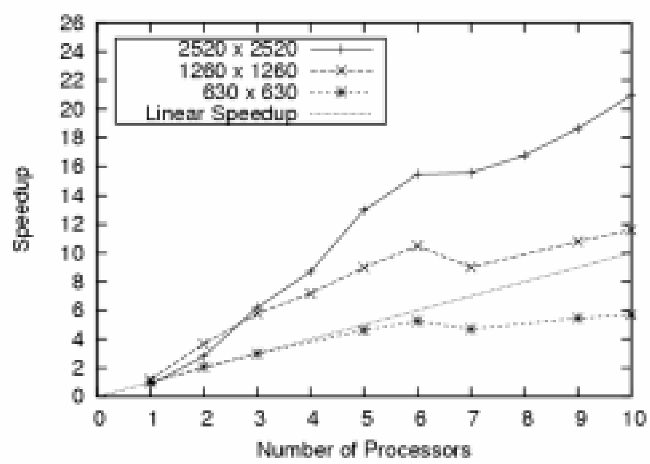
- **Runtime** of the experiments for 3 different system sizes:



Dedicated cluster: speedup

- **Speedup** with respect to the sequential program for 3 different system sizes:

$$\text{speedup} = \frac{\text{runtime of the sequential version}}{\text{runtime of the parallel version}}$$



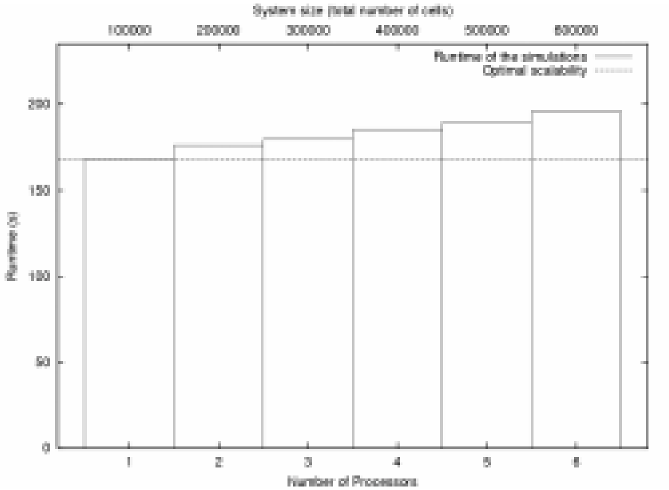
Analysis of execution

- High **computation-to-communication ratio** (~10 for slaves):

Dedicated cluster: Scalability

- An application is said to be **scalable** if:
  - When the **number of processors** and the **problem size** are increased by the same factor, the **running time** remains **the same**

Results:



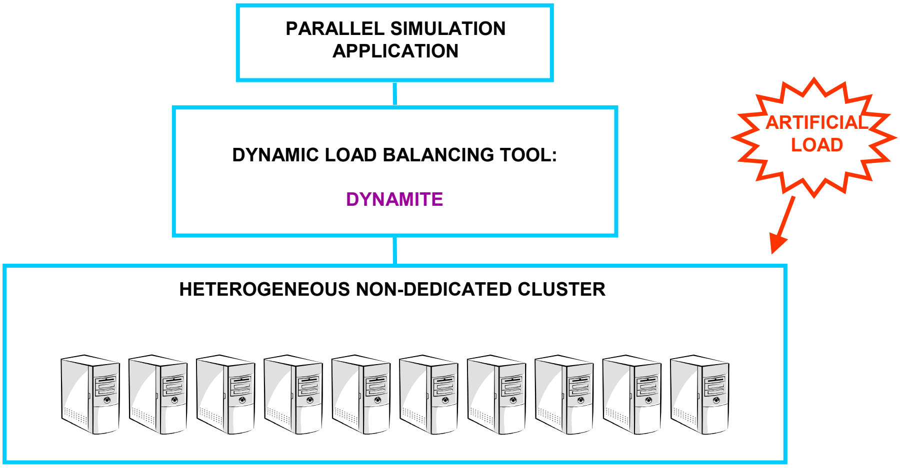
  - **Small overhead: 2% to 5%** ⇒ **good scalability for small clusters**

Non-dedicated cluster

- **Two main differences** respect to most previous implementations of CA-based models:

  1. **Modular approach**: the model is executed on top of a dynamic load balancing tool
     - More flexibility than directly implementing the load balancing on the application
  2. It is possible to migrate load to **cluster nodes initially not belonging to the pool**

Non-dedicated cluster



PARALLEL SIMULATION APPLICATION

DYNAMIC LOAD BALANCING TOOL:

DYNAMITE

HETEROGENEOUS NON-DEDICATED CLUSTER

ARTIFICIAL LOAD

Dynamic load balancing

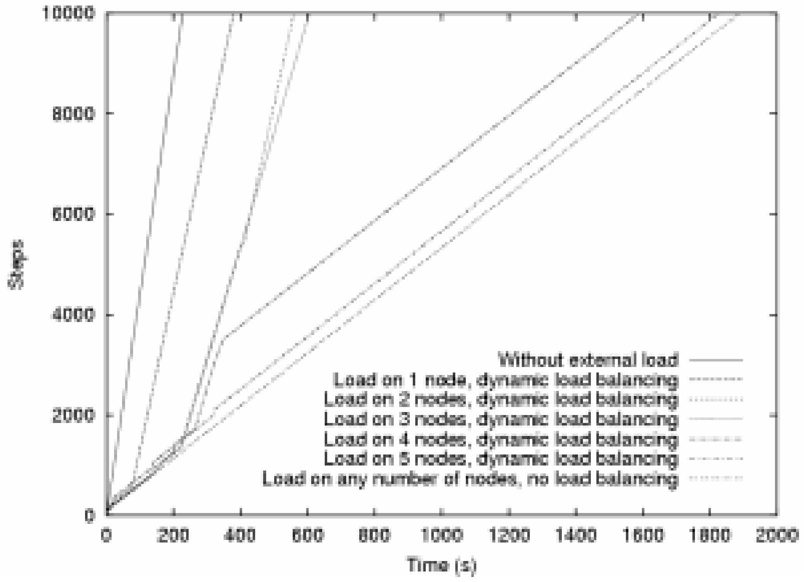
- Dynamic load balancing tool:
  - **DYNAMITE:**
    - Developed by the group: “Section Computational Science”, from the University of Amsterdam
    - Based on “Dynamic PVM”, a re-implementation of PVM that adds dynamic load balancing
    - It **monitors the use of the cluster nodes** (CPU, memory)
    - It **dynamically migrates tasks** when one of them gets over- or under-used, as defined by configurable threshold values
    - http://www.science.uva.nl/research/scs/Software/dynamite

Non-dedicated cluster: Performance

- **Experiments**:
  - Parallel application: 6 computing nodes + 1 master node
  - Artificial load: from 0 to 5 nodes
  - 10 nodes available on the cluster
- **Results**:

CONFIGURATION	EXECUTION TIME (s)	IMPROVEMENT (RATIO)	IMPROVEMENT (PERCENT)
No load balancing with artificial load on 1-5 nodes	1895.08	---	---
Load balancing with load on 1 node	384.59	4.93	80 %
Load balancing with load on 2 nodes	564.76	3.36	70 %
Load balancing with load on 3 nodes	611.12	3.10	68 %
Load balancing with load on 4 nodes	1595.75	1.19	16 %
Load balancing with load on 5 nodes	1833.82	1.03	3 %
No load, with or without load balancing	233.43	---	---

Execution progress for different load levels



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Conclusions (1)

- **CA model for laser dynamics simulation → alternative to differential equations**
  1. **It reproduces the phenomenology** of laser dynamics
     - Threshold pumping, laser behaviours (constant regime, laser spiking, irregular oscillations), dependence on parameters
  2. **Flexible and robust model**:
     - Modified model reproduces the behaviour of pulsed pumped lasers
  3. **New methodological approach**:
     - **Laser** as a **complex system**
     - **Applications**: when rate equations → difficult to integrate or not applicable
     - **Parallel model**

Conclusions (2)

4. **Parallel implementation** using the message passing paradigm

5. **Dedicated clusters**:
   - Implementation takes a **good advantage of parallelization**:
     - In spite of frequent communications (CA model only partially uncoupled)
     - High computation-to-communication ratio
   - **Good performance**
   - **Good scalability** on small clusters

6. **Heterogeneous non-dedicated clusters**:
     - **Dynamic load balancing**
     - Artificial load

     } Performance improvement if free nodes
   - **It is feasible to run a parallel version of the model**

Future work

- **Simulations of specific laser systems**:
  - 3D model
  - Boundary conditions

- **Scalability** of the model for **massive paralellism**

- **Implementation using Grid Computing**:
  - **Desktop grid** implementation already done → Using BOINC system
  - Application selected by the **EDGES Project**:

    ("Enabling Desktop Grids for e-Science",
    7th Framework Program project, European Union)

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Breve resumen en español

Modelado y Simulación de la Dinámica del Láser con Autómatas Celulares en Computadores Paralelos y Distribuidos

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Objetivos

1. Modelar un láser utilizando un autómata celular que reproduzca la fenomenología de su dinámica.
2. Aprovechar la naturaleza intrínsecamente paralela de los autómatas celulares para desarrollar una implementación paralela eficiente de dicho modelo, sobre computadores paralelos y distribuidos:
   - Cluster dedicado
   - Cluster heterogéneo no dedicado

Conclusiones (1)

- **Modelo de autómata celular (AC) para simular la dinámica láser → alternativa a las ecuaciones diferenciales**
  1. **Reproduce la fenomenología** de la dinámica láser:
     - Bombeo umbral, comportamientos del láser (régimen constante, oscilaciones de relajación o "laser spiking", oscilaciones irregulares), dependencia respecto a los parámetros del sistema
  2. **Modelo flexible y robusto**:
     - Un modelo modificado reproduce el comportamiento de los láseres de bombeo pulsado
  3. **Nuevo enfoque metodológico**:
     - **Láser** como un **sistema complejo**
     - **Aplicaciones**: cuando las ecuaciones de balance del láser → difíciles de integrar o no aplicables
     - **Modelo intrínsecamente paralelo**

Conclusiones (2)

4. **Implementación paralela** usando el paradigma de paso de mensajes

5. **Clusters dedicados**:
   - La implementación **aprovecha la paralelización**:
     - A pesar de las comunicaciones frecuentes (el modelo de AC es sólo parcialmente desacoplado)
     - Alta tasa computación-comunicación
   - **Buen rendimiento**.
   - **Buena escalabilidad** sobre clusters pequeños.

6. **Clusters heterogéneos no dedicados**:
     - **Balanceo dinámico de la carga**
     - Carga artificial

     } Mejora del rendimiento si hay nodos libres
   - **Es factible ejecutar una versión paralela del modelo**