Negative refraction from balanced quasi-planar chiral inclusions

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March 6, 2007

Abstract

This work proposes a quasi-planar chiral resonator suitable for the design of negative refractive index matamaterials. An analytical model is presented for determining the metamaterial polarizabilities, which is the basis of a further study of the the viability of negative refraction in chiral and racemic arrangements of inclusions made up with the proposed quasi-planar chiral resonator . The present analysis is expected to pave the way for the design and building of feasible negative refractive index matamaterials whose inclusions can be manufactured by means of standard photo-etching techniques.

Key words: Negative refraction, matamaterials.

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1 Introduction

The main aim of this work is to explore the possibility of obtaining negative refraction in a medium made of a random arrangement of quasi-planar chiral inclusions. Artificial bi-isotropic chiral media made of random arrangements of metallic chiral inclusions are known since long, after the works of K. Lindmann [1]. More recently, a mixture of such kind of inclusions was proposed in [2] as a way to obtain negative refractive index metamaterials, a proposal further developed in [3] and [4]. Negative refraction in a mixture of chiral inclusions and resonant wires was also analyzed in [5]. The main advantage of using chiral elements to provide negative refraction is that only one kind of inclusions is necessary to obtain simultaneous negative values of ϵ and μ. If the design of this particle was *quasi-planar*, a key additional advantage would come from the possibility of using conventional printed-circuit fabrication techniques to manufacture such inclusions.

2 The proposed inclusion

The inclusion here proposed is shown in Fig. 1. It is basically a broadside-coupled version of the two-turn spiral resonator already proposed by some of the authors in [6]. The analysis in that paper shows that the proposed element can be characterized by a quasi-static LC circuit, where L is the inductance of a single ring with the same radius and width as the inclusion, and $C = 2\pi r C_{\text{pul}}$ is the total capacitance between the rings. There are, however, two main differences between the structure of Fig. 1 and the 2-SR analyzed in [6]. First, due to the broadside coupling of the proposed inclusion, the distributed capacitance between the rings can be very large, thus reducing the electrical size of the inclusion near the resonance. Second, when the element is excited near the resonance, there will appear strong magnetic and electric dipoles oriented parallel to the resonator axis. The electric dipole comes from the strong electric field between the upper and lower rings that appears near the resonance.

Neglecting losses, and following the analysis in [6], the circuit equation for the total current in the element (i.e., for the sum of the currents excited on both rings, which must be angle-independent [6]) is given by

$$\left(\frac{1}{j\omega C} + j\omega L\right) I = \mathcal{E} \; , \tag{1}$$

where $\mathcal{E}$ represents the external voltage excitation, which can be expressed as

$$\mathcal{E} = \begin{cases} -j\omega\pi r 2 B_z^{\text{ext}} \; , & \text{magnetic excitation} \\ t\, C_0/C\, E_z^{\text{ext}} \; , & \text{electric excitation} \end{cases} \tag{2}$$

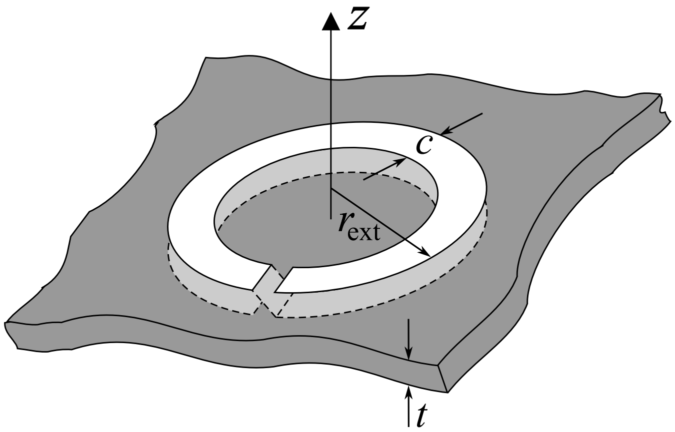

Figure 1: The proposed inclusion is formed by two identical conducting rings etched on both sides of a dielectric substrate and connected by a via in order to obtain an helicoidal shape.

where r is the mean radius of the inclusion, B_z^{ext} the z-component of the external magnetic field, t the substrate thickness, E_z^{ext} the z-component of the external electric field, and C_0 the total capacitance between the rings in the absence of the dielectric substrate. (The factor C_0/C is introduced because when a parallel-plate capacitor is excited by a normal external field, the electric field inside the capacitor is merely the external one multiplied by the above factor). From the above equations, the electric and magnetic moments excited in the inclusion when subjected to external electric and/or magnetic fields can be obtained following the same procedure already reported in [7]:

$$m_z = \alpha_{zz}^{\text{mm}} B_z^{\text{ext}} - \alpha_{zz}^{\text{em}} E_z^{\text{ext}} \quad (3)$$

$$p_z = \alpha_{zz}^{\text{ee}} E_z^{\text{ext}} + \alpha_{zz}^{\text{em}} B_z^{\text{ext}} , \quad (4)$$

where

$$\alpha_{zz}^{\text{mm}} = \frac{\pi^2 r4}{L}\left(\frac{\omega_0 2}{\omega 2} - 1\right)^{-1} \quad (5)$$

$$\alpha_{zz}^{\text{em}} = \pm j\pi r^2 t C_0 \frac{\omega_0 2}{\omega}\left(\frac{\omega_0 2}{\omega} - 1\right)^{-1} \quad (6)$$

$$\alpha_{zz}^{\text{ee}} = t2 C_0^2 L \frac{\omega_0 4}{\omega 2}\left(\frac{\omega_0 2}{\omega 2} - 1\right)^{-1} , \quad (7)$$

with $\omega_0 = \sqrt{1/LC}$ being the resonance frequency. From (5)–(7) it is found the following useful identity:

$$\alpha_{zz}^{\mathrm{mm}}\alpha_{zz}^{\mathrm{ee}} = -\left(\alpha_{zz}^{\mathrm{em}}\right)2\,. \tag{8}$$

It should be noted that the inclusion also presents non-resonant electric polarizabilities in the transverse z-plane α_{xx}^{ee} and α_{yy}^{ee} [7]. However since these polarizabilities are almost constant with frequency and not very large, they can be neglected in a first approximation.

When N chiral inclusions are randomly assembled, the resulting medium becomes bi-isotropic with constitutive relations given by

$$\mathbf{D} = \varepsilon_0(1+\chi_e)\mathbf{E} + j\sqrt{\varepsilon_0\mu_0}\,\kappa\mathbf{H} \tag{9}$$

$$\mathbf{B} = -j\sqrt{\varepsilon_0\mu_0}\,\kappa\mathbf{E} + \mu_0(1+\chi_m)\mathbf{H}\,. \tag{10}$$

The electric, χ_e, magnetic, χ_m, and cross, κ, susceptibilities are related to the inclusion polarizabilities through

$$\chi_e = \frac{N}{\Delta\varepsilon_0}\frac{\alpha_{zz}^{\mathrm{ee}}}{3} \tag{11}$$

$$\chi_m = \frac{N\mu_0}{\Delta}\frac{\alpha_{zz}^{\mathrm{mm}}}{3} \tag{12}$$

$$\kappa = \pm j\frac{N}{\Delta}\sqrt{\frac{\mu_0}{\varepsilon_0}}\frac{\alpha_{zz}^{\mathrm{em}}}{3}\,, \tag{13}$$

where the factor $1/3$ arises from the random arrangement and Δ is a common factor that depends on the homogenization procedure. If Lorentz local field theory is used for the determination of the local field seen by each inclusion, and (8) is taken into account, this factor is given by

$$\Delta = 1 - \frac{N}{9}\left\{\frac{\alpha_{zz}^{\mathrm{ee}}}{\varepsilon_0} + \mu_0\alpha_{zz}^{\mathrm{mm}}\right\}\,. \tag{14}$$

From (8) and (11)–(13) it follows that

$$\chi_e(\omega)\chi_m(\omega) = [\kappa(\omega)]^2\,. \tag{15}$$

3 Negative refraction

As it is well known, the general dispersion equation for plane waves in lossless chiral media is

$$k = \pm k_0\left(\sqrt{(1+\chi_e)(1+\chi_m)} \pm \kappa\right)\,, \tag{16}$$

where $k_0 = \omega\sqrt{\epsilon_0\mu_0}$ is the free-space wavenumber. The four solutions of (16) correspond to right- and left-hand circularly polarized waves, depending on the sign of κ. In order to avoid complex solutions of (16), and therefore forbidden frequency bands for plane wave propagation, it would be desirable that $\chi_e(\omega) = \chi_m(\omega)$. According to (15) this implies that

$$\chi_e(\omega) = \chi_m(\omega) = |\kappa(\omega)|\,, \tag{17}$$

namely, the electric and magnetic responses of the inclusions should be very similar [4].

The general conditions for backward-wave propagation in chiral media were analyzed in [8], and can be summarized as

$$\sqrt{\epsilon_r\mu_r} \pm \kappa < 0\,, \tag{18}$$

where the negative sign of the square root has to be chosen if ϵ_r and μ_r are both negative. According to (18), if $\kappa 2 > |\epsilon_r\mu_r|$ only one of the solutions of (16) can be a backward-wave and therefore will experience negative refraction at the interface with ordinary media. This is indeed the case when (17) is satisfied and both χ_e and χ_m are negative. In such case, negative refraction will take place for only one of the eigenmodes of (16) provided that $\chi_e = \chi_m = |\kappa| < -0.5$. This condition is less restrictive than the condition found for ordinary media (for instance, for a balanced mixture of inclusions of opposite helicity): $\chi_e,\, \chi_m\, < -1$. The price to pay for this bandwidth enlargement is that only one of the solutions of (16) shows negative refraction. This scenario is illustrated in Fig. 2, where an incident linearly polarized wave is considered.

Returning to the inclusions, from (11)–(13) it is found that condition (17) is satisfied provided that

$$c_0^2\alpha_{zz}^{\text{ee}}(\omega) = \alpha_{zz}^{\text{mm}}(\omega) = \pm j c_0 \alpha_{zz}^{\text{em}}(\omega)\,, \tag{19}$$

where c_0 is the velocity of light in vacuum. In principle this condition is compatible with (5)–(7), which make it possible to find particular designs that satisfy this condition by using the analytical expressions for L and C_{pul} reported in [7]. A substrate with permittivity similar to free space (a foam for instance) has been chosen in order to simplify computations. With this substrate ($\epsilon = \epsilon_0$), a suitable design is: width of the strips $c = 2\,\text{mm}$, external radius $r_{\text{ext}} = r + c/2 = 5\,\text{mm}$, and separation between strips $t = 2.35\,\text{mm}$. Following [7], the resonance frequency of the proposed configuration should be about $2.3\,\text{GHz}$, which gives an electrical size for the inclusion ($\sim \lambda/13$) that is acceptable for a practical metamaterial design.

In order to validate our analytical results, the electric and magnetic polarizabilities of the inclusions have been numerically computed following the procedure

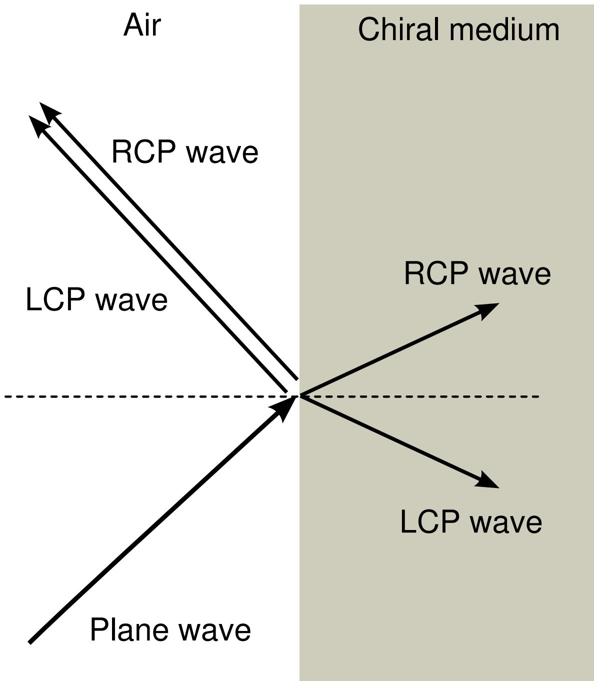

Figure 2: Illustration of the negative refraction of a linearly polarized wave at the interface with a chiral metamaterial made of inclusions such as that shown in Fig. 1. Only one of the two eigenwaves that can propagate in the chiral medium shows negative refraction while the reflected wave is elliptically polarized.(**RCP, LCP tienen que ser definidos aqui**)

described in [9]. This procedure mainly consists in placing the particle inside a TEM waveguide and to determine the polarizabilities from the loaded waveguide's reflection and transmission coefficients (see [9] for more details). The results for the meaningful quantities $\mu_0\alpha_{zz}^{\rm mm}$ and $\alpha_{zz}^{\rm ee}/\epsilon_0$ are shown in Fig. 3, which clearly confirm the predictions of our analytical model.

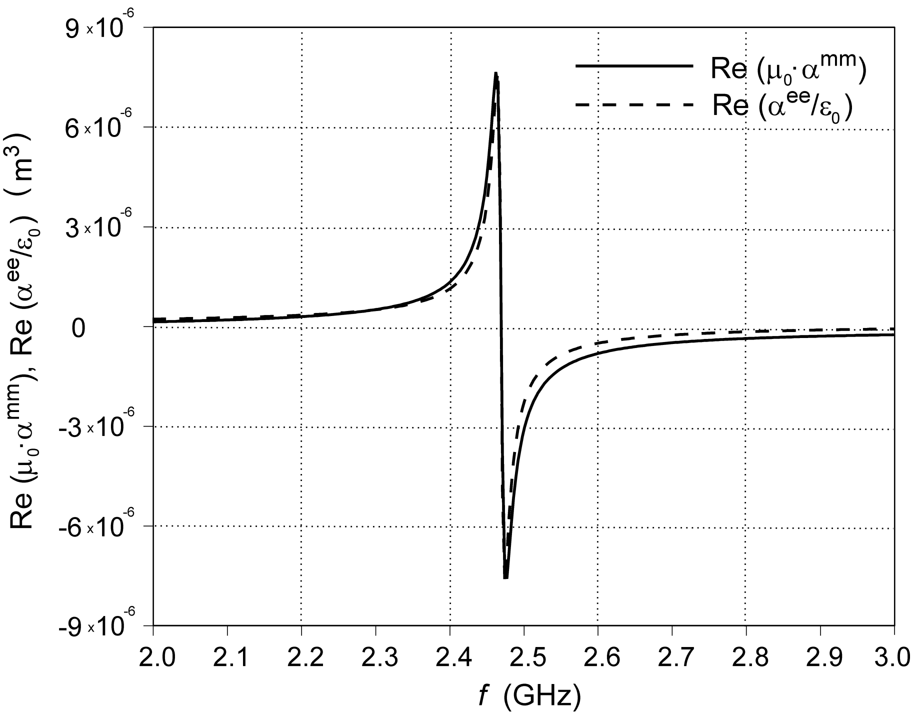

Figure 3: Numerical computation of $\mu_0\alpha_{zz}^{\rm mm}$ and $\alpha_{zz}^{\rm ee}/\epsilon_0$ for the inclusion shown in Fig.1. Width of the strips $c = 2\,$mm, external radius $r_{\rm ext} = r + c/2 = 5\,$mm, and separation between strips $t = 2.35\,$mm.

The frequency bandwidth for negative refraction in a metamaterial made of a random arrangement of the proposed inclusions can be computed taking the electric susceptibility χ_e of such medium from (11) with Δ given by (14). The dimensions and characteristics of the inclusions are as previously reported and the number of inclusions per unit volume is $N = (12)^{-3}\,{\rm mm}^{-3}$. Both the analytical and the numerical results obtained from the data in Fig. 3 are shown in Fig. 4. From the previous analysis and numerical results directly follows that the curves (not shown) for the magnetic, χ_m, and cross, κ, susceptibilities must be quite similar. Although Fig. 4 shows some differences between the analytical and numerical results, the qualitative agreement is apparent. In both cases, a significant negative refraction frequency band appears for both the random and the racemic mixtures. As already mentioned, such frequency bands are limited by the straight lines $\chi_e = -0.5$ and $\chi_e = -1$ respectively (see Fig. 4).

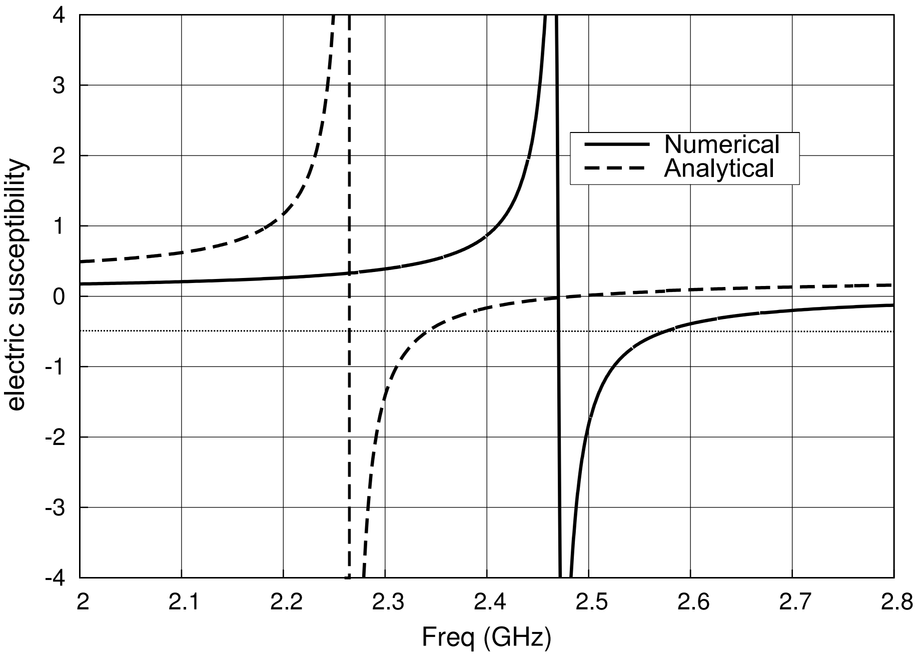

Figure 4: Analytical and numerical results for the electric susceptibility χ_e of a random arrangement of chiral inclusions as those shown in Fig. 1. The parameters of the inclusions are given in the text and are the same as in Fig. 3. The average volume per inclusion is $V = 12^3$ mm3.

Since the proposed inclusions show a balanced electric and magnetic response (that is, (19) is satisfied), it follows from (17) and (16) that there are no forbidden frequency bands for plane wave propagation in the considered chiral medium. This implies that the transition from backward to forward propagation occurs through a point of zero phase velocity and non-zero group velocity, a fact that recalls the behavior of balanced right/left-handed transmission lines reported in [10]. Needless to say, a similar behavior would appear in racemic mixtures of the proposed inclusions, where the propagation constant for plane waves is given by $k = \pm k_0 \sqrt{(1+\chi_e)(1+\chi_m)}$

4 Conclusion

The feasibility of negative refractive index metamaterials made of a random arrangement of balanced chiral quasi-planar inclusions has been analyzed. It has been proposed an specific design susceptible of being easily manufactured by means of the standard photo-etching techniques. The proposed design has been found to provide the necessary behavior for all the resonant polarizabilities in order to produce a significant negative refractive index bandwidth near the resonance. Finally, it has been shown that a behavior quite similar to the previously reported balanced right/left-handed transmission lines can be achieved in three-dimensional arrangements of the proposed inclusions.

Acknowledgments

This work has been supported by the Spanish Ministry of Education and Science by project contracts TEC2004–04249–C02–02 and TEC2004-03214.

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