

Electrically small isotropic three-dimensional magnetic resonators for metamaterial design

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Abstract

The problem of the design of artificial magnetic resonators for isotropic metamaterials is addressed. The internal symmetries that ensure an isotropic behavior of such resonators are analyzed and some specific designs based on the proper arrangement of modified split ring resonators are proposed. These proposals are validated by electromagnetic simulations and experiments. The reported results are likely to have applications in the design of devices such as negative refractive index materials, super-lenses, and meta-surfaces with isotropic response

After the first practical demonstration of a medium having simultaneously negative permittivity and permeability, also called *left-handed* medium (LHM) [1], a great wave of interest was generated in the scientific community due to the unique physical properties of such LHM [2, 3]. Once the feasibility of such media was shown, practical isotropic two-dimensional designs based on well known printed circuit technologies came soon [4–6]. However, the practical implementation of a truly isotropic LHM still remains an open question. Isotropic artificial media with negative permittivity can be made, in principle, by assembling a three dimensional array of connected wires [7–9]. Proposals for isotropic artificial media having negative permeability include cubic arrays of *Split Ring Resonators* (or SRRs; see Fig.1.a) [10, 11] and *Omega* particles [12], as well as isotropic spherical combinations of SRRs [13]. More recently, a cubic array of modified SRRs and connected wires has been proposed for the implementation artificial LHM [14]. Regarding cubic arrays of SRRs and *Omega* particles [10, 12], the couplings between these elements should be investigated before postulating isotropy. Regarding spherical combinations of SRRs, it is well known that they only provide isotropy in two dimensions [13]. Finally, the recently proposed cubic array of modified SRRs and wires [14] clearly provides an isotropic medium, since the unit cell is invariant under all the transformations of the group of symmetry of a cube; i.e. the group generated by the transformations $\{I, -I, C_{4x}, C_{4y}, C_{4z}\}$ (or the O_h group in Schonflies notation). However, the proposed structure is difficult to implement in practice. First of all, very high permittivity dielectrics are involved in its design. Secondly, at difference from other previously proposed combinations of SRRs and metallic strips [4], it is made of bulk metallic parts that can not be manufactured by using standard printed circuit technologies. The aim of this letter is to propose and analyze some practical ways to artificial negative permeability media made by using modified SRR particles and standard printed circuit technologies (eventually, such structures could be combined with printed strips to produce LHM). First, the minimum group of symmetry providing isotropic behavior will be investigated. Then, it will be shown through experiments that some previous proposals lacking this symmetry do not produce an isotropic response. Finally, some practical proposals possessing such symmetry will be analyzed.

To begin with, let us consider the sub-group of symmetry generated by the two non-commutative products of 90° rotations around two coordinate axes: $\{I, C_{4x} \cdot C_{4y}, C_{4y} \cdot C_{4x}\}$ (i.e. the T group in Schonflies notation). After some cumbersome but straightforward

calculations, it is readily shown that any second order tensor invariant by this group of symmetry should be, in fact, a scalar. Nevertheless, this fact does not guarantee that the medium can be characterized by some scalar permittivity ϵ and permeability μ . In fact, magnetoelectric couplings (see, for instance [15]) may cause the medium to be bi-isotropic, or *chiral* [16]. To ensure that the medium is not a chiral medium, it must be invariant by the inversion, $-I$, [17]. Thus, to ensure both isotropy and non-chirality, the medium basic units should remain invariant by the symmetry group $\{I, -I, C_{4x} \cdot C_{4y}, C_{4y} \cdot C_{4x}\}$ (or the T_h group in Schonflies notation).

To illustrate the effects of breaking this symmetry, we will consider the basic unit shown in the inset of Fig.2. It is a cubic array of SRRs, where the SRRs are arranged in such a way that the whole array satisfies inversion symmetry, so that chirality is forbidden. It has been shown that, if the couplings among the SRRs are neglected, such basic unit has an isotropic response [11]. However, the analyzed basic unit is not invariant by the $\{I, -I, C_{4x} \cdot C_{4y}, C_{4y} \cdot C_{4x}\}$ group of symmetry. In particular, it is not invariant after the transformation $C_{4x} \cdot C_{4y}$, as it can be easily realized by inspection. In fact, it can be also realized by inspection that it is impossible to build up a cubic combination of SRRs invariant by the aforementioned group of symmetry. The basic unit shown in Fig.2 has been fabricated and placed at different orientations in a rectangular waveguide, in order to measure its response. Before measuring the whole structure, the frequencies of resonance of the different SRRs forming the structure were measured. All of them were found to be inside the interval $f = (2.321 \pm 0.002)$ GHz. No other resonances appeared in the TE₁₀ monomode range of the utilized waveguide, i.e. from 1.7 to 2.6 GHz. As it can be seen in the figure, the analyzed basic unit presents an anisotropic response, which can not be justified by tolerances in the SRRs fabrication process. This experiment shows that couplings between adjacent elements destroy the isotropy when the basic unit has not the appropriate symmetry, as in this case. Splitting of the original resonance of single SRRs into several resonances is also observed. Similar experimental results were obtained for the cubic array of *Omega* particles proposed in [12]. In that case the response also change with the orientation, and the resonance is split in four resonances.

In order to overcome the aforementioned difficulties, a modification of the previous structure was analyzed. This modification substitutes the SRRs by the non-bianisotropic SRRs reported in [17], and shown in Fig.1.b. The resulting structure, which is shown in the inset of

Fig.3, is invariant under the symmetry group $\{I, C_{4x} \cdot C_{4y}, C_{4y} \cdot C_{4x}\}$, without the inversion. The isotropy of the response has been checked by experiments similar to those reported in Fig.2. The results are shown in Fig.3, where an isotropic behavior can be clearly seen. As it was already mentioned, the analyzed structure has not inversion symmetry. Therefore, media made with these basic units may show bi-isotropy. In order to check this possibility the rotation of the plane of polarization of the light by this structure has been investigated experimentally. In the experiment, the structure was placed inside a rectangular waveguide and the cross-polarized transmission coefficient was measured by using appropriate detectors (two orthogonal input and output dipole antennas). The result is also shown Fig.3, where a chiral behavior (expressed in the rotation of the plane of polarization of the transmitted wave) is clearly observed. Interestingly, it is known that each non-bianisotropic SRR forming the analyzed structure does not show bi-anisotropy by itself [17]. Therefore, the chirality of the structure comes from the couplings between the SRRs forming the array.

If chirality was not desired, the design must be invariant under the whole group of symmetry $\{I, -I, C_{4x} \cdot C_{4y}, C_{4y} \cdot C_{4x}\}$, including inversion. Two magnetic resonators having this symmetry are shown in Fig.4. Fig.4.a shows an spherical arrangement of two-splits SRRs, which is a modification of the spherical resonator proposed in [13]. Fig.4.b shows a cubic array of broadside-coupled double-split SRRs (the design of each SRR is a modification of a previous proposal reported in [15]). This structure has the same symmetries as the resonator shown in Fig.4.a. In order to illustrate the isotropy of the structure proposed in Fig.4.a, its response when it is placed inside a rectangular waveguide has been simulated by using the commercial electromagnetic solver *CST Microwave Studio*. The results of this simulation are shown in Fig.5. The same transmission coefficient was obtained for all the simulated orientations (the small differences can be attributed to numerical tolerances of the simulation process). Similar results are expected for the structure of Fig.4.b, since it has the same symmetry as the analyzed spherical resonator.

In summary, the possibility of designing isotropic three-dimensional magnetic resonators by properly arranging modified SRRs has been verified. It has been also verified that the couplings between the different SRRs can not be ignored in such design. Therefore, the array must show the appropriate structural symmetry in order to achieve an isotropic response. It has been shown that the group of symmetry generated by the transformations $\{I, C_{4x} \cdot C_{4y}, C_{4y} \cdot C_{4x}\}$ (the T group in Schonflies notation) is enough to ensure an isotropic response of

the design. However, if a non bi-isotropic response is desired (for instance, in order to design an effective medium fully characterized by some scalars ϵ and μ), the inversion symmetry must be added to this group (thus forming the T_h group in Schonflies notation). Starting from this analysis, some specific designs of bi-isotropic and non bi-isotropic resonators have been proposed and tested by experiments and numerical simulations.

Acknowledgments

This work has been supported by the Spanish Ministry of Education and Science under project contract TEC2004-04249-C02-02, and by the Grant Agency of Czech Republic under project 102/03/0449. Authors also thank to Esperanza Rubio for manufacturing the resonators used in the experiments.

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List of Figure Captions

- **Fig.1:** (a) Split Ring Resonator (SRR). (b) Non-bianisotropic SRR.
- **Fig.2:** Plots of the transmission coefficient ($|S_{21}|$) of a rectangular waveguide with a cubic array of SRRs (see the inset) placed inside at different orientations. The waveguide is a rectangular R22 waveguide, with dimensions 109×55 mm². The dimensions of the SRRs are (see Fig.1) $r_{ext} = 7$ mm, $w = 1.25$ mm, $d = 0.5$ mm, and $g = 1$ mm. The substrate where the SRRs were printed is ARLONC 250-LX-0193-43-11, with dielectric constant $\epsilon_r = 2.43$ and thickness $t = 0.49$ mm. The size of the cube is $a = 20$ mm. Solid line corresponds to the cube oriented with the z -axis along the waveguide axis, and the y -axis along the electric field of the fundamental TE₁₀ waveguide mode (the only one excited in the experiment). Dashed and dash-dot lines corresponds to rotations C_{8y} , and $C_{8y} \cdot C_{8z}$ of the cube, respectively. Due to the couplings between different SRRs, the original resonance frequency of each single SRR, $f = 2.321 \pm 0.002$ GHz, changes and splits into several resonances depending on the orientation.
- **Fig.3:** Plots of the transmission coefficient ($|S_{21}|$) of a rectangular waveguide with a cubic array of SRRs (see the inset) placed inside at different orientations. The waveguide size, the dimensions of the SRR, and the substrate characteristics are as in Fig.1 (the frequency of resonance of the isolated non-bianisotropic SRRs was found to be in the interval $f_0 = 2.385 \pm 0,002$ GHz). The analyzed orientations are the same as in Fig.1. The transmission coefficient for the cross-polarized wave is also shown.
- **Fig.4:** a) Spherical isotropic magnetic resonator. b) Isotropic cubic array of broadside-coupled SRRs possessing the same symmetries as the structure shown in (a).
- **Fig.5:** Plots of the transmission coefficient ($|S_{21}|$) for the spherical resonator shown in Fig.4.a placed inside a rectangular waveguide at different orientations. The waveguide is a standard R9 waveguide with dimensions 218×124 mm². The dimensions of the resonator are $r_{ext} = 20$ mm, $w = 4$ mm, $d = 3$ mm, $g = 8$ mm. The dielectric filling the space between the broadside-coupled strips has $\epsilon_r = 4$. Solid line corresponds to the resonator with the z -axis along the waveguide axis, and the y -axis along the

electric field of the fundamental TE_{10} waveguide mode. Dashed and dash-dotted lines corresponds to rotations C_{4y} , and $C_{8y} \cdot C_{8x}$ of the resonator, respectively.

Figure 1

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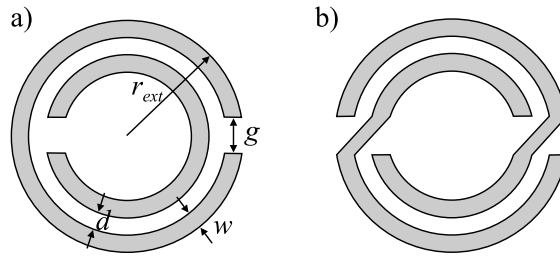


Figure 2

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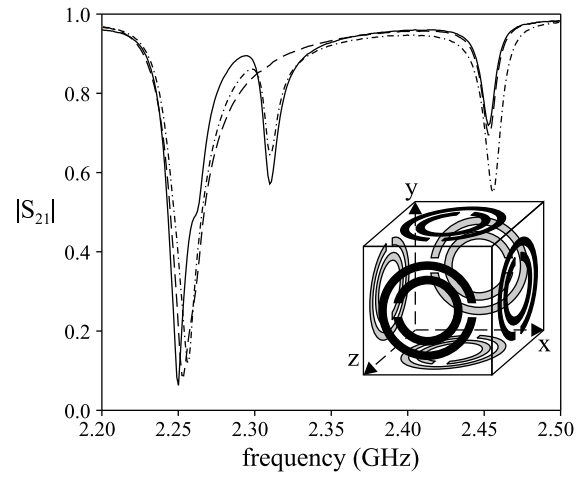


Figure 3

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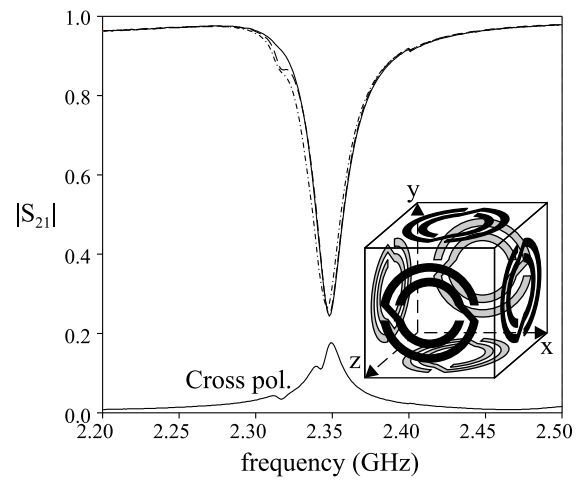


Figure 4

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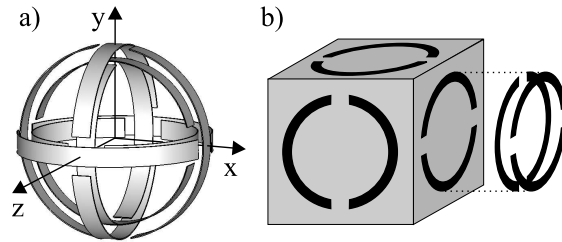


Figure 5

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